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Two-loop Yang-Mills theory in the world-line formalism and an Euler-Heisenberg type action

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Abstract

Within the framework of the world-line formalism we write down in detail a two-loop Euler-Heisenberg type action for gluon loops in Yang-Mills theory and discuss its divergence structure. We exactly perform all the world-line moduli integrals at two loops by inserting a mass parameter, and then extract divergent coefficients to be renormalized.

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1 Introduction

The Bern-Kosower method is described as a set of simple rules to obtain gluon scattering amplitudes at one loop , and it is known to improve the computational efficiency over the current Feynman diagram technique [1]. Those rules are derived from a string theory in the limit where the inverse string tension vanishes, as a consequence of the idea that a string world-sheet degenerates into a desired particle diagram at a singular point on the boundary of moduli space [2]-[4]. The integration over moduli space naturally covers all necessary Feynman diagrams appearing in field theory, and we hence have a compact master formula for particle scattering amplitudes. Thus the most conspicuous point in this formalism is that the diagram summation is already finished in the formula without introducing the loop integral and the Dirac trace for a given scattering [5, 6]. This idea is also applied to graviton scattering [7].

The discovery of the Bern-Kosower rules has also stimulated investigations for a new mathematical structure of quantum field theory; how to reflect the string-like structure into field theory as such. The first rederivation of the Bern-Kosower rules was accomplished by Strassler in Ref. [8], where the background field method, the proper time method and the path integral method for a first quantized $0+1$ dimensional field theory (world-line formulation) are well combined [9]. There are many other fruitful examples along this stream [10] (see Section 1 of Ref. [11] for updated references), and these examples are the strong incentives to study the world-line formalism from the theoretical point of view, especially from the viewpoint of its higher loop extensions [12] . The present paper also discusses two-loop Yang-Mills theory in the world-line formalism with the aim to develop techniques for higher loops.

This paper is a continuation of the previous work [11], where the effective action of Yang-Mills theory at the two-loop order is derived based on the world-line formalism; also developed there is a certain technique, which generates multiloop generalizations of the one-loop trace-log (determinant) formula. However, these arguments are still inside the shell of formal arguments, since we are left with the problem of how to deal with the multi-integrals of world-line moduli parameters. The moduli integrals of a higher genus world-sheet are too complicated to perform, while one might naturally expect that this situation would be improved in the field theory limit. Although we, of course, have an option to computerize these complicated integrals, there are still difficulties, for example in three loop QED integrals [13].

It is certainly valuable to analyze two-loop integrals of Yang-Mills theory in the world-line

formalism in particular if many outer particles or an Euler-Heisenberg type constant field are involved. It might also hint to the world-line moduli integrations at higher loop orders.

In this paper, after a detailed derivation of the two-loop Euler- Heisenberg action with gluon loops in a pseudo-abelian gauge field background, we shall present some technical issues of how to deal with the world-line moduli integrals in the gluon effective action at the second order of the Taylor expansion in terms of external background fields. This analysis is also essential to examine the divergence structure related to a wave function and gauge fixing parameter renormalization. We only discuss the gluon loop part, since the ghost loop part is rather simple and can be dealt with in the same way as the gluon loop case.

When we perform the integrals, we insert a mass parameter in order to regularize divergences. Generally speaking, massive propagators in the Feynman rule method are difficult to integrate in an analytic way. Contrastingly in our formalism, we shall go through the entire procedure analytically, and all the results will be written in hypergeometric functions. This is certainly an intriguing point of this paper.

This paper is organized as follows. In Section 2, we write down our starting formulae for the gluon loop effective action at two loops. We slightly modify a few notations from the previous presentation [11] through the path and proper time inversions presented in Appendix A. In Section 3, a derivation of the one-loop β-function coefficient serves as an example for how calculations in our formalism can be simplified by specializing to the pseudo-abelian case. In Section 4, we apply the pseudo-abelian technique to the calculation of the gluon effective action presented in Section 2. Here, we only perform the world-line path integral parts. This Section is a completion of the parts outlined in the previous paper [11], and the details of the computation are contained in Appendix B. In Section 5, we further study how to integrate the world-line moduli integral parts, which are the final integrations to obtain a fully integrated form of Euler-Heisenberg type action. For simplicity, we only consider the gluon kinetic term, through the Taylor expansions concerning the external field strength. The Taylor coefficients are the functions of world-line moduli parameters, and we show that these coefficients can be integrated; the details are in Appendices C and D. Appendix E is the Feynman diagram analysis to be compared with our results.

2 Two-loop effective action

Let us first review in brief the world-line representation of the two-loop effective action in

Yang-Mills theory [11]. In this paper we only discuss the gluon loop part. It is given by

$$\Gamma[A] = I_1[A] + I_2[A] \ , \tag{2.1}$$

where ($I_1 = \Gamma_1 + \Gamma_2$, $I_2 = \Gamma_3^{(2)}$ in the previous paper)

$$\begin{aligned} I_1[A] \ &= \ -\frac{1}{8}\int_0^\infty dS \int_0^S d\tau_\alpha \int_0^\infty dT_3 \oint [\mathcal{D}x]_S \int_{w(0)=x(0)}^{w(T_3)=x(\tau_\alpha)} [\mathcal{D}w]_{T_3} \\ &\quad \times \Big[(\dot{w}_\mu(0) - \dot{x}_\mu(0))\, \dot{w}_\rho(T_3)\; W^{ae}_{\mu[\sigma\rho]\nu}[x; S, \tau_\alpha, 0] \\ &\qquad + \dot{x}_\nu(0)\, \dot{w}_\rho(T_3)\; W^{ae}_{\mu[\sigma\rho]\mu}[x; S, \tau_\alpha, 0] \Big]\; W^{ea}_{\sigma\nu}[w; T_3, 0] \ , \end{aligned} \tag{2.2}$$

and

$$\begin{aligned} I_2[A] \ &= \ \frac{1}{4}\int_0^\infty dT_1\, dT_2 \oint [\mathcal{D}x_1]_{T_1} \oint [\mathcal{D}x_2]_{T_2}\; \delta^4(x_1^\mu(0) - x_2^\mu(0)) \\ &\qquad \times \mathrm{Tr}_C\Big[\lambda^a W_{\mu\nu}[x_1; T_1, 0]\Big]\; \mathrm{Tr}_C\Big[\lambda^a W_{\nu\mu}[x_2; T_2, 0]\Big] \ , \end{aligned} \tag{2.3}$$

with the following compact notations:

$$\int [\mathcal{D}x]_T\, F[x] = \int \mathcal{D}x\, e^{-\frac{1}{4}\int_0^T \dot{x}^2\, d\tau} F[x] \qquad \text{for any functional } F[x] \ , \tag{2.4}$$

$$W^{ea}_{\sigma\nu}[w; T_3, 0] = \mathrm{P}\exp\left\{ \int_0^{T_3} d\tau M_\tau[w] \right\}^{ea}_{\sigma\nu} \ , \tag{2.5}$$

$$W^{ae}_{\mu\sigma\rho\nu}[x; S, \tau_\alpha, 0] = \mathrm{Tr}_C\left[\lambda^a\, \mathrm{P}\exp\left\{ \int_{\tau_\alpha}^{S} d\tau M_\tau[x] \right\}_{\mu\sigma} \lambda^e\, \mathrm{P}\exp\left\{ \int_0^{\tau_\alpha} d\tau M_\tau[x] \right\}_{\rho\nu} \right] \ , \tag{2.6}$$

and

$$(M_\tau[x])^{ab}_{\mu\nu} = 2i\left[F^c_{\mu\nu}(x(\tau')) - \frac{1}{2}\delta_{\mu\nu} A^c_\rho(x(\tau')) \cdot \partial_{\tau'} x^\rho(\tau') \right]_{\tau'=\tau} (\lambda^c)^{ab} \ . \tag{2.7}$$

Here we have slightly changed the notations used in the previous paper [11]; (i) the previous definition of M is associated with $D = \partial - iA$, while the present M_τ is associated with $D = \partial + iA$, (ii) the path ordering directions are modified to be the standard one, and some related formulae are listed in Appendix A. As in the previous paper, we always use Euclidean space-time conventions.

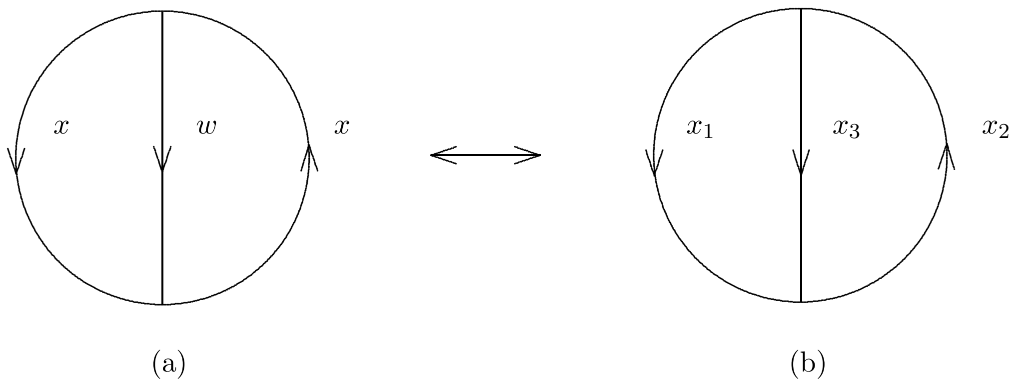


Figure 1: (a) The loop type parametrization. (b) The symmetric parametrization.

It is convenient to have another representation for $I_1[A]$, based on the symmetric parametrization in Figure 1(b), which treats the individual gluon lines in a more equal way, thus allowing a greater class of transformations by inverting and relabelling the gluon paths and leading to significant simplifications in the concrete calculations. The expression (2.2) is based on the loop type parametrization in Figure 1(a). The transformation rules between (a) and (b) are known [12, 14], and we thus have the following symmetric representation for $I_1[A]$:

$$
\begin{aligned}
I_1[A] = -\frac{1}{8}\int_0^\infty dT_1 dT_2 dT_3 \int d^D y_1 \int d^D y_2 \left[\prod_{k=1}^{3} \int_{x_k(0)=y_1}^{x_k(T_k)=y_2} [\mathcal{D}x_k]_{T_k}\right] \\
\times \Big[\left(\dot{x}_{3\mu}(0) - \dot{x}_{1\mu}(0)\right) \dot{x}_{3\rho}(T_3)\, \tilde{W}^{ae}_{\mu[\sigma\rho]\nu}[x_2^{-1}, x_1; T_2, 0, T_1, 0] \\
+ \dot{x}_{1\nu}(0)\, \dot{x}_{3\rho}(T_3)\, \tilde{W}^{ae}_{\mu[\sigma\rho]\mu}[x_2^{-1}, x_1; T_2, 0, T_1, 0] \Big] W^{ea}_{\sigma\nu}[x_3; T_3, 0] \ ,
\end{aligned} \tag{2.8}
$$

where

$$
T_1 = \tau_\alpha \ , \qquad T_2 = S - \tau_\alpha \ , \tag{2.9}
$$

and

$$
\tilde{W}^{ae}_{\mu\sigma\rho\nu}[x_2^{-1}, x_1; T_2, 0, T_1, 0] = \mathrm{Tr}_C \left[\lambda^a \, \mathrm{P}\exp\left\{ \int_0^{T_2} d\tau M_\tau[x_2^{-1}] \right\}_{\mu\sigma} \lambda^e \, \mathrm{P}\exp\left\{ \int_0^{T_1} d\tau M_\tau[x_1] \right\}_{\rho\nu} \right] . \tag{2.10}
$$

Note that the transition from loop type to symmetric parametrization requires both splitting the loop path into two parts and inverting one of them (which we denote by x_2^{-1}) to achieve all three paths to start at y_1 and to end at y_2. In contrast to the naive expectation, this suggests that in a general background, one may not just write down the product of three propagators starting

and ending at identical points to represent a loop with inserted propagator. See Appendix A for a detailed definition of the notation x_2^{-1} and some comments on path inversion

After finishing the next section, we shall discuss the world-line path integrals for x_k in Section 4, and the integrals of the world-line moduli parts (proper times) T_k in Section 5.

3 The pseudo-abelian case

For the rest of this paper, we confine ourselves to the pseudo-abelian su(2) with constant field strength. Thus we assume

$$A^a_\mu(x) = \mathcal{A}_\mu(x)\, n^a \qquad \text{with} \quad n^a n^a = 1 \quad \text{and constant } n \ , \tag{3.1}$$

i.e. the color dependence of the non-abelian gauge fields is supposed to be factored out in form of a constant unit vector in color space. Within these settings calculations are simplified considerably, though non-abelian results can still be reproduced as shall be seen below. As an example and to introduce some notations, we here show a brief sketch of how our formalism works within the calculation of one-loop β-function coefficients.

The assumption (3.1) leads to similar decompositions for the field strength

$$F^a_{\mu\nu}(x) = \mathcal{F}_{\mu\nu}(x)\, n^a \quad \text{with} \quad \mathcal{F}_{\mu\nu}(x) = \partial_\mu \mathcal{A}_\nu(x) - \partial_\nu \mathcal{A}_\mu(x) \tag{3.2}$$

and the matrix $M_\tau[x]$

$$M_\tau[x] = \mathcal{M}_\tau[x] \otimes (n^c \lambda^c) \equiv \mathcal{M}_\tau[x] \otimes \mathcal{T}_- \ , \tag{3.3}$$

where $\mathcal{M}_\tau[x]$ is the Lorentz matrix defined by

$$\mathcal{M}_\tau[x] = 2i\Big[\mathcal{F}(x) - \frac{1}{2}\,\mathcal{A}_\rho(x)\,\dot{x}_\rho\,\mathbf{1_L}\Big] \tag{3.4}$$

and $\mathbf{1}_L$ denotes the unit Lorentz matrix (in the Euclidean space).

So far we have not used our additional assumption of a constant field strength, nor have we fixed the gauge for the external gauge fields A^a_μ. If we take into account the constancy of the field strength, we may choose

$$\mathcal{A}_\mu(x) = \frac{1}{2} x_\nu \mathcal{F}_{\nu\mu} \ , \tag{3.5}$$

henceforth expecting $\mathcal{M}_\tau[x]$ to be of the form

$$\mathcal{M}_\tau[x] = 2i\Big[\mathcal{F} - \frac{1}{4}\, x_\sigma \mathcal{F}_{\sigma\rho}\, \dot{x}_\rho\, \mathbf{1}_L\Big], \tag{3.6}$$

rather than (3.4). In addition, it is convenient to define the integrated matrices (omitting the index τ), and we can simply write

$$\int_0^T M_\tau[x]d\tau = \mathcal{M}[x] \otimes \mathcal{T}_- \qquad \text{with} \quad \mathcal{M}[x] = \int_0^T \mathcal{M}_\tau[x]d\tau \ . \tag{3.7}$$

It is the benefit of confining ourselves to the pseudo-abelian case, that the $M_\tau[x]$ matrices for different values of the parameter τ become commuting quantities

$$[\, M_\tau[x],\, M_{\tau'}[x]\,] = 0 \ . \tag{3.8}$$

Thus we are allowed to drop path ordering from all of our expressions. This leads to the following decomposition

$$\mathrm{P}\exp\{\mathcal{M} \otimes \mathcal{T}_-\} = \mathbf{1}_L \otimes \mathcal{I} \ + \ \sinh\{\mathcal{M}\} \otimes \mathcal{T}_- \ + \ \cosh\{\mathcal{M}\} \otimes \mathcal{T}_+ \ , \tag{3.9}$$

in terms of the $su(2)$ matrices

$$\mathcal{T}_- = n^c\lambda^c \ , \quad \mathcal{T}_+ = (\mathcal{T}_-)^2 \ , \quad \mathcal{I} = \mathbf{1}_C - \mathcal{T}_+ \ , \qquad \text{where} \quad \mathbf{1}_C = diag(1,1,1) \ . \tag{3.10}$$

Now, using the properties

$$\mathrm{Tr}_C\, \mathcal{T}_- = 0, \qquad C_A \equiv \mathrm{Tr}_C\, \mathcal{T}_+ = 2, \qquad \mathrm{Tr}_C\, \mathcal{I} = 1 \tag{3.11}$$

the one-loop effective action for the gluon loop is calculated as follows [15]:

$$\begin{aligned}
\Gamma_G^{1-loop}[A] &= -\frac{1}{2}\int_0^\infty \frac{dT}{T} e^{-m^2T} \oint [\mathcal{D}x]_T \left(\mathrm{P}\exp\int_0^T M_\tau[x]\, d\tau\right)^{aa}_{\mu\mu} \\
&= -\frac{1}{2}\int_0^\infty \frac{dT}{T} e^{-m^2T} \oint [\mathcal{D}x]_T \ (D + C_A \mathrm{Tr}_L(\cosh\mathcal{M})) \\
&= -\frac{1}{4}\int_0^\infty \frac{dT}{T} e^{-m^2T} \oint [\mathcal{D}x]_T \left(D + C_A \mathrm{Tr}_L(e^{\mathcal{M}})\right) \quad + \quad (\mathcal{F} \to -\mathcal{F}) \ ,
\end{aligned} \tag{3.12}$$

where we have introduced the gluon mass term e^{-m^2T} for regularization. The second contribution with $\mathcal{F}$ replaced by $-\mathcal{F}$ counts for a factor of two, thus with the one-loop path integral normalization

$$\oint \mathcal{D}x \ \exp\Big\{-\frac{1}{4}\int_0^T d\tau \left[\dot{x}^2 + 2ix\mathcal{F}\dot{x}\right]\Big\} = (4\pi T)^{-D/2} \mathrm{det}_L^{-1/2}\Big(\frac{\sin\mathcal{F}T}{\mathcal{F}T}\Big) \int d^D x_0 \tag{3.13}$$

we are led to

$$\Gamma_G^{1-loop}[A] = \frac{-1}{2(4\pi)^{D/2}} \int_0^\infty dT T^{-1-D/2} e^{-m^2T} \left[D + C_A \mathrm{Tr}_L (e^{2i\mathcal{F}T}) \det{}_L^{-1/2} \Big(\frac{\sin \mathcal{F}T}{\mathcal{F}T} \Big) \right] \int d^D x_0 \ . \quad (3.14)$$

For now we are interested in the two-point function only, i.e. in the second functional derivative of the effective action with respect to A^a_μ. To this end, we only need the second order term of an expansion of (3.14) in terms of $\mathcal{F}$. Omitting constant and higher order terms we find

$$\Gamma_G^{1-loop}[A] = \ \cdots - \ \frac{C_A}{2(4\pi)^{D/2}} \Big(\frac{D}{12} - 2 \Big) \int_0^\infty dT\, T^{1-D/2} \, e^{-m^2T} \int d^D x_0 \, \mathrm{Tr}_L \mathcal{F}^2 \ + \cdots \ . \quad (3.15)$$

Setting $D = 4 - 2\varepsilon$ and performing the T integration leads to the following pole structure in ε:

$$\Gamma_G^{1-loop}[A] = \ \cdots - \ \frac{g_0^2 C_A}{(4\pi)^2} \left(\frac{-10}{3\varepsilon} \right) \int d^D x_0 \left(-\frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}_{\mu\nu} \right) + \mathcal{O}(\varepsilon^0) \ + \cdots \ , \quad (3.16)$$

where we have revived the gauge coupling g and where g_0 is the dimensionless coupling constant defined by $g = g_0 \mu^\varepsilon$. Finally we calculate the functional derivative and transform into momentum space: using

$$\int d^D x_0 \left(-\frac{1}{4} \mathcal{F}_{\rho\sigma} \mathcal{F}_{\rho\sigma} \right) \ \longrightarrow \ -\delta^{ab} (\delta_{\mu\nu} k^2 - k_\mu k_\nu) \quad (3.17)$$

we read off

$$\Pi_{G\,\mu\nu}^{ab} = -\frac{g_0^2 C_A \delta^{ab}}{(4\pi)^2} \left(\frac{10}{3\varepsilon} \right) (\, \delta_{\mu\nu} k^2 - k_\mu k_\nu \,) + \mathcal{O}(\varepsilon^0) \ . \quad (3.18)$$

Similarly, the one-loop contribution from a ghost loop can be calculated: As can be deduced from Ref. [11], the ghost one-loop action is given by changing the overall normalization in (3.12) from 1/2 to -1, and only employing the Lorentz scalar term in (2.7); i.e., define $-iA^c_\mu \dot{x}_\mu \lambda^c \equiv \tilde{M}_\tau$ instead of using M_τ. The corresponding pseudo-abelian quantity $\tilde{\mathcal{M}}$ is defined analogically to the gluon loop case (q.v. Eqs. (3.6) and (3.7)). Thus we find the one-loop ghost action

$$\begin{aligned} \Gamma_{FP}^{1-loop}[A] &= \int_0^\infty \frac{dT}{T} e^{-m^2T} \oint [\mathcal{D}x]_T \Big(\mathrm{P} \exp \int_0^T \tilde{M}_\tau[x] \, d\tau \Big)^{aa} \\ &= \int_0^\infty \frac{dT}{T} e^{-m^2T} \oint [\mathcal{D}x]_T \ \Big(1 + C_A \cosh \tilde{\mathcal{M}} \Big) \\ &= \frac{1}{2} \int_0^\infty \frac{dT}{T} e^{-m^2T} \oint [\mathcal{D}x]_T \ \Big(1 + C_A e^{\tilde{\mathcal{M}}} \Big) \quad + \quad (\mathcal{F} \to -\mathcal{F}) \ . \end{aligned} \quad (3.19)$$

Again taking into account the $-\mathcal{F}$ term by a factor of two and using (3.13), we arrive at

$$\Gamma_{FP}^{1-loop}[A] = \frac{1}{(4\pi)^{D/2}} \int_0^\infty dT\, T^{-1-D/2} e^{-m^2T} \left[1 + C_A \det{}_L^{-1/2} \Big(\frac{\sin \mathcal{F}T}{\mathcal{F}T} \Big) \right] \int d^D x_0 \ . \quad (3.20)$$

Expanding this expression in the same way as done in (3.15), we derive

$$\Gamma^{1-loop}_{FP}[A] = \cdots + \frac{C_A}{12(4\pi)^{D/2}} \int_0^\infty dT\, T^{1-D/2}\, e^{-m^2T} \int d^Dx_0\, \mathrm{Tr}_L \mathcal{F}^2 \;+\cdots \tag{3.21}$$

$$= \cdots - \frac{g_0^2 C_A}{(4\pi)^2}\left(\frac{-1}{3\varepsilon}\right) \int d^Dx_0 \left(-\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu}\right) + \mathcal{O}(\varepsilon^0) \;+\cdots\;, \tag{3.22}$$

and hence

$$\Pi_{FP}{}^{ab}_{\mu\nu} = -\frac{g_0^2 C_A \delta^{ab}}{(4\pi)^2}\left(\frac{1}{3\varepsilon}\right)(\,\delta_{\mu\nu}k^2 - k_\mu k_\nu\,) + \mathcal{O}(\varepsilon^0)\;. \tag{3.23}$$

Gathering Eqs. (3.18) and (3.23), the correct (one-loop) β-function coefficient 11/3 is reproduced.

4 Two-loop Euler-Heisenberg formulas

Now, let us consider the extension of the above calculations to the two-loop case. We deal with the symmetric representations (2.3) and (2.8). In the two-loop case , as understood from (2.7), we have to keep in mind that the sign of the $x\mathcal{F}\dot{x}$ term changes due to the τ derivative, if we invert the path. For example in (2.10), one should notice (see also Appendix A) that

$$\begin{aligned}\left(M_\tau[x_2^{-1}]\right)^{ab}_{\mu\nu} &= 2i\left[F^c_{\mu\nu}(x_2^{-1}(\tau')) - \frac{1}{2}\delta_{\mu\nu}A^c_\rho(x_2^{-1}(\tau'))\cdot\partial_{\tau'}x^{-1}_{2\rho}(\tau')\right]_{\tau'=\tau}(\lambda^c)^{ab}\\ &= 2i\left[F^c_{\mu\nu}(x_2(\tau')) + \frac{1}{2}\delta_{\mu\nu}A^c_\rho(x_2(\tau'))\cdot\partial_{\tau'}x_{2\rho}(\tau')\right]_{\tau'=T_2-\tau}(\lambda^c)^{ab}\\ &= \left(M_{T_2-\tau}[x_2]\right)^{ba}_{\nu\mu}\;.\end{aligned} \tag{4.1}$$

Reflecting this fact, it is rather convenient to introduce the signature index κ $(=\pm1,0)$ on the Lorentz matrix $\mathcal{M}$:

$$\mathcal{M}^{(\kappa)}_k \overset{\text{def.}}{=} \int_0^{T_k} d\tau\, 2i\Big[|\kappa|\mathcal{F} - \frac{1}{4}\,\kappa\, x_{k\sigma}\mathcal{F}_{\sigma\rho}\,\dot{x}_{k\rho}\,\mathbf{1}_L\Big]\;, \tag{4.2}$$

where the k stands for the line labels 1,2 and 3. With this matrix notation (4.2), the general form for the color matrix part of the action (2.8) is written in the form

$$\begin{aligned}&\tilde{W}^{ae}_{\gamma\delta\alpha\beta}[x_2^{-1},x_1;T_2,0,T_1,0]\,W^{ea}_{\sigma\nu}[x_3;T_3,0]\\ &= \mathrm{Tr}_C\Big[\lambda^a \exp\Big\{\mathcal{M}^{(-)}_2\otimes\mathcal{T}_-\Big\}_{\gamma\delta}\lambda^e \exp\Big\{\mathcal{M}^{(+)}_1\otimes\mathcal{T}_-\Big\}_{\alpha\beta}\Big]\exp\Big\{\mathcal{M}^{(+)}_3\otimes\mathcal{T}_-\Big\}^{ea}_{\sigma\nu}\;.\end{aligned} \tag{4.3}$$

After using the expansion (cf. Eq. (3.9))

$$\exp\Big\{\mathcal{M}^{(\kappa)}_k\otimes\mathcal{T}_-\Big\} = \mathbf{1}_L\otimes\mathcal{I} \;+\; \sinh\Big\{\mathcal{M}^{(\kappa)}_k\Big\}\otimes\mathcal{T}_- \;+\; \cosh\Big\{\mathcal{M}^{(\kappa)}_k\Big\}\otimes\mathcal{T}_+\;, \tag{4.4}$$

we perform the color traces applying the following formulae:

$$\begin{aligned}
\mathrm{Tr}_C\,(\lambda^a \mathcal{T}_\pm\, \lambda^e \mathcal{T}_\pm)\, \mathcal{I}^{ea} &= 2 \\
\mathrm{Tr}_C\,(\lambda^a \mathcal{T}_\pm\, \lambda^e \mathcal{I})\, \mathcal{T}^{ea}_\pm &= 2 \\
\mathrm{Tr}_C\,(\lambda^a \mathcal{I}\, \lambda^e \mathcal{T}_\pm)\, \mathcal{T}^{ea}_\pm &= \pm 2\ ,
\end{aligned} \tag{4.5}$$

where the 3rd formula follows from the 2nd one with the properties

$$(\mathcal{T}_\pm)^T = \pm \mathcal{T}_\pm, \qquad \mathcal{I}^T = \mathcal{I}\ . \tag{4.6}$$

For any other combinations of A, B and C chosen out of $\{\mathcal{I},\ \mathcal{T}_-,\ \mathcal{T}_+\}$, the following formula applies

$$\mathrm{Tr}_C\,(\lambda^a A\, \lambda^e B)\, C^{ea} = 0\ . \tag{4.7}$$

Thus the quantity (4.3) is calculated as follows:

$$\begin{aligned}
&\tilde{W}^{ae}_{\gamma\delta\alpha\beta}[x_2^{-1}, x_1; T_2, 0, T_1, 0]\, W^{ea}_{\sigma\nu}[x_3; T_3, 0] \\
&= \quad 2\,\delta_{\alpha\beta}\Big[\cosh\Big\{\mathcal{M}_2^{(-)}\Big\}_{\gamma\delta} \cosh\Big\{\mathcal{M}_3^{(+)}\Big\}_{\sigma\nu} +\ \sinh\Big\{\mathcal{M}_2^{(-)}\Big\}_{\gamma\delta} \sinh\Big\{\mathcal{M}_3^{(+)}\Big\}_{\sigma\nu}\Big] \\
&\quad + 2\,\delta_{\gamma\delta}\Big[\cosh\Big\{\mathcal{M}_1^{(+)}\Big\}_{\alpha\beta} \cosh\Big\{\mathcal{M}_3^{(+)}\Big\}_{\sigma\nu} -\ \sinh\Big\{\mathcal{M}_1^{(+)}\Big\}_{\alpha\beta} \sinh\Big\{\mathcal{M}_3^{(+)}\Big\}_{\sigma\nu}\Big] \\
&\quad + 2\,\delta_{\sigma\nu}\Big[\cosh\Big\{\mathcal{M}_1^{(+)}\Big\}_{\alpha\beta} \cosh\Big\{\mathcal{M}_2^{(-)}\Big\}_{\gamma\delta} +\ \sinh\Big\{\mathcal{M}_1^{(+)}\Big\}_{\alpha\beta} \sinh\Big\{\mathcal{M}_2^{(-)}\Big\}_{\gamma\delta}\Big] \qquad (4.8) \\
&= \quad \exp\Big\{\mathcal{M}_1^{(0)}\Big\}_{\alpha\beta} \exp\Big\{\mathcal{M}_2^{(-)}\Big\}_{\gamma\delta} \exp\Big\{\mathcal{M}_3^{(+)}\Big\}_{\sigma\nu} \\
&\quad + \exp\Big\{\mathcal{M}_1^{(+)}\Big\}_{\alpha\beta} \exp\Big\{\mathcal{M}_2^{(0)}\Big\}_{\gamma\delta} \exp\Big\{-\mathcal{M}_3^{(+)}\Big\}_{\sigma\nu} \\
&\quad + \exp\Big\{\mathcal{M}_1^{(+)}\Big\}_{\alpha\beta} \exp\Big\{\mathcal{M}_2^{(-)}\Big\}_{\gamma\delta} \exp\Big\{\mathcal{M}_3^{(0)}\Big\}_{\sigma\nu} \quad + \quad (\mathcal{F} \to -\mathcal{F})\ . \qquad (4.9)
\end{aligned}$$

Now, as sketched in Ref. [11], performing the trivial integral (the 1st term) in Eq.(4.2)

$$\begin{aligned}
&\exp\Big\{\int_0^{T_k} d\tau\big(-\frac{1}{4}\,\dot{x}_k^2\big)\Big\} \exp\Big\{\pm \mathcal{M}_k^{(\kappa)}\Big\}_{\eta\xi} \\
&\quad = \quad \exp\{\pm 2i\,|\kappa|\,T_k\,\mathcal{F}\}_{\eta\xi} \exp\Big\{-\frac{1}{4}\int_0^{T_k} d\tau[\dot{x}_k^2 + 2i\,(\pm\kappa)\, x_{k\sigma}\mathcal{F}_{\sigma\rho}\,\dot{x}_{k\rho}]\Big\}\ ,
\end{aligned} \tag{4.10}$$

and introducing the quantities

$$S^{(\kappa_1,\kappa_2,\kappa_3)} = -\frac{1}{4}\sum_{k=1}^{3}\int_0^{T_k} d\tau\Big[\,\dot{x}_k^2 + 2i\,\kappa_k\, x_{k\sigma}\mathcal{F}_{\sigma\rho}\,\dot{x}_{k\rho}\,\Big]\ , \qquad (\kappa_a = \pm 1, 0) \tag{4.11}$$

we have the following formula:

$$
\begin{aligned}
&\exp\Big\{-\frac{1}{4}\sum_{k=1}^{3}\int\limits_0^{T_k} d\tau\, \dot{x}_k^2\Big\}\, \tilde{W}^{ae}_{\gamma\delta\alpha\beta}[x_2^{-1}, x_1; T_2, 0, T_1, 0]\, W^{ea}_{\sigma\nu}[x_3; T_3, 0] \\
&= \quad \delta_{\alpha\beta}\ \exp\Big\{2i\, T_2\, \mathcal{F}\Big\}_{\gamma\delta} \exp\Big\{2i\, T_3\, \mathcal{F}\Big\}_{\sigma\nu} \quad e^{S^{(0,-,+)}} \\
&\quad + \delta_{\gamma\delta}\ \exp\Big\{2i\, T_1\, \mathcal{F}\Big\}_{\alpha\beta} \exp\Big\{2i\, T_3\, \mathcal{F}\Big\}_{\nu\sigma} \quad e^{S^{(+,0,-)}} \\
&\quad + \delta_{\sigma\nu}\ \exp\Big\{2i\, T_1\, \mathcal{F}\Big\}_{\alpha\beta} \exp\Big\{2i\, T_2\, \mathcal{F}\Big\}_{\gamma\delta} \quad e^{S^{(+,-,0)}} \quad + \quad (\mathcal{F} \to -\mathcal{F})\ .
\end{aligned} \tag{4.12}
$$

It is worth noticing here that the interaction terms $x_k \mathcal{F} \dot{x}_k$ defined on the three different lines possess different κ values; this fact is related to the $su(2)$ structure ε^{abc}. In the following, we calculate I_1 and I_2 separately, since these two quantities involve different world-line topology.

4.1 The $I_1[A]$ part

Applying the formula (4.12) to Eq. (2.8), we obtain the following expressions (For the convenience of presentation, we split $I_1[\mathcal{A}]$ into two quantities depending on whether $x_1 x_3$ or $x_3 x_3$ correlations.):

$$
I_1[\mathcal{A}] = \Gamma_1[\mathcal{A}] + \Gamma_2[\mathcal{A}]\ , \tag{4.13}
$$

where

$$
\begin{aligned}
\Gamma_1[\mathcal{A}] = &-\frac{1}{8}\int\limits_0^{\infty} dT_1 dT_2 dT_3 \int d^D y_1 \int d^D y_2 \left[\prod_{k=1}^{3} \int\limits_{x_k(0)=y_1}^{x_k(T_k)=y_2} \mathcal{D}x_k\right] \dot{x}_3^{\mu}(0)\ \dot{x}_3^{\rho}(T_3) \\
&\times \Bigg[\Big[e^{2iT_2\mathcal{F}} \Big(e^{2iT_3\mathcal{F}} - \mathbf{1}_L \mathrm{Tr}_L\Big(e^{2iT_3\mathcal{F}}\Big)\Big)\Big]_{\mu\rho}\ e^{S^{(0,-,+)}} \\
&+ \Big[e^{2iT_2\mathcal{F}} \Big(e^{-2iT_1\mathcal{F}} - \mathbf{1}_L \mathrm{Tr}_L\Big(e^{-2iT_1\mathcal{F}}\Big)\Big)\Big]_{\mu\rho}\ e^{S^{(+,-,0)}} \\
&+ \Big[e^{-2i(T_1+T_3)\mathcal{F}} - \mathbf{1}_L \mathrm{Tr}_L(e^{-2i(T_1+T_3)\mathcal{F}})\Big]_{\mu\rho}\ e^{S^{(+,0,-)}} \Bigg] \quad + \quad (\mathcal{F} \to -\mathcal{F})\ ,
\end{aligned} \tag{4.14}
$$

$$
\begin{aligned}
\Gamma_2[\mathcal{A}] = &-\frac{1}{8}\int\limits_0^{\infty} dT_1 dT_2 dT_3 \int d^D y_1 \int d^D y_2 \left[\prod_{k=1}^{3} \int\limits_{x_k(0)=y_1}^{x_k(T_k)=y_2} \mathcal{D}x_k\right] \dot{x}_1^{\mu}(0)\ \dot{x}_3^{\rho}(T_3) \\
&\times \Bigg[\Big[-2i\sin(2\mathcal{F}(T_2+T_3)) - e^{2i(T_2-T_3)\mathcal{F}} + e^{2iT_2\mathcal{F}} \mathrm{Tr}_L e^{2iT_3\mathcal{F}}\Big]_{\mu\rho}\ e^{S^{(0,-,+)}} \\
&+ \Big[-2i\sin(2\mathcal{F}(T_1+T_2)) - e^{2i(T_2-T_1)\mathcal{F}} + e^{2iT_2\mathcal{F}} \mathrm{Tr}_L e^{2iT_1\mathcal{F}}\Big]_{\mu\rho}\ e^{S^{(+,-,0)}} \\
&+ \Big[-2\cos(2\mathcal{F}(T_1+T_3)) + e^{2i(T_3-T_1)\mathcal{F}} + \mathbf{1}_L \mathrm{Tr}_L e^{2i(T_1+T_3)\mathcal{F}}\Big]_{\mu\rho}\ e^{S^{(+,0,-)}} \Bigg] \\
&+ \quad (\mathcal{F} \to -\mathcal{F})\ .
\end{aligned} \tag{4.15}
$$

Then we perform the path integrals of the form

$$< \dot{x}^\mu_a(\tau)\ \dot{x}^\nu_b(\tau') >_{(\kappa_1,\kappa_2,\kappa_3)} = \int d^D y_1 \int d^D y_2 \left[\prod_{k=1}^{3} \int_{x_k(0)=y_1}^{x_k(T_k)=y_2} \mathcal{D}x_k\right] \dot{x}^\mu_a(\tau)\ \dot{x}^\nu_b(\tau')\ e^{S^{(\kappa_1,\kappa_2,\kappa_3)}}\ , \tag{4.16}$$

and this yields

$$< \dot{x}^\mu_a(\tau)\ \dot{x}^\nu_b(\tau') >_{(\kappa_1,\kappa_2,\kappa_3)} = \mathcal{N}^{(\kappa_1,\kappa_2,\kappa_3)} \partial_\tau \partial_{\tau'} \mathcal{G}^{ab}_{\mu\nu}(\tau,\tau';\kappa_1,\kappa_2,\kappa_3)\ , \tag{4.17}$$

where

$$\mathcal{N}^{(\kappa_1,\kappa_2,\kappa_3)} = (4\pi)^{-D}\, \mathrm{det}_L^{-1/2}\Big(\sum_{l=1}^{3} \kappa_l \mathcal{F} \cot \kappa_l \mathcal{F} T_l\Big) \prod_{k=1}^{3} T_k^{-D/2}\, \mathrm{det}_L^{-1/2}\left(\frac{\sin \kappa_k \mathcal{F} T_k}{\kappa_k \mathcal{F} T_k}\right) \int d^D x_0\ , \tag{4.18}$$

and [1]

$$\begin{aligned} \mathcal{G}^{ab}_{\mu\nu}(\tau,\tau';\kappa_1,\kappa_2,\kappa_3) &= -\delta_{ab} G^a_{\mu\nu}(\tau,\tau') \\ &+2\Big([\sum_{k=1}^{3} \kappa_k \mathcal{F} \cot(\kappa_k \mathcal{F} T_k)]^{-1}\Big)_{\rho\sigma} \Big(\frac{e^{2i\kappa_a \mathcal{F}\tau}-1}{e^{2i\kappa_a \mathcal{F} T_a}-1} - \frac{1}{2}\Big)_{\mu\rho} \Big(\frac{e^{2i\kappa_b \mathcal{F}\tau'}-1}{e^{2i\kappa_b \mathcal{F} T_b}-1} - \frac{1}{2}\Big)_{\nu\sigma}, \end{aligned} \tag{4.19}$$

with

$$G^a_{\mu\nu}(\tau,\tau') = \begin{cases} \delta_{\mu\nu} G^a_B(\tau,\tau') = \delta_{\mu\nu}\Big[\, |\tau-\tau'| - \dfrac{(\tau-\tau')^2}{T_a}\,\Big] & (\kappa_a = 0) \\ \Big[\dfrac{1}{2\mathcal{F}^2}\Big(\dfrac{\mathcal{F}}{\sin(\mathcal{F}T_a)} e^{-i\kappa_a \mathcal{F} T_a \partial_\tau G^a_B(\tau,\tau')} + i\kappa_a \mathcal{F} \partial_\tau G^a_B(\tau,\tau') - \dfrac{1}{T_a}\Big)\Big]_{\mu\nu} & (\kappa_a \neq 0)\ . \end{cases} \tag{4.20}$$

Inserting each value of (4.17) at $(\tau,\tau') = (0, T_3)$ into Eqs. (4.14) and (4.15), we therefore obtain (the details are presented in Appendix B)

$$\begin{aligned} I_1[\mathcal{A}] = -\frac{1}{2}\,(4\pi)^{-D} \int_0^\infty dT_1 dT_2 dT_3\ \mathrm{det}_L^{1/2}\left(\frac{\mathcal{F}^2}{\Delta_\mathcal{F}}\right) \Big\{ \\ \mathrm{Tr}_L\Bigg(\frac{\mathcal{F}^2 T_3}{\Delta_\mathcal{F} \sin \mathcal{F}T_2} \Big[\, 2\sin \mathcal{F}T_1 \cos 2\mathcal{F}(T_1+2T_2) - 2\sin \mathcal{F}(T_1+T_2) \cos \mathcal{F}(2T_1+3T_2) \\ + \{\, 1 - 2\cos 2\mathcal{F}(T_1+T_2)\,\} \sin \mathcal{F}T_2 \cos \mathcal{F}(T_1-T_2)\Big] \\ + \frac{\mathcal{F}}{\Delta_\mathcal{F}} \Big[\, 4\sin \mathcal{F}T_1 \sin \mathcal{F}T_2 \sin 2\mathcal{F}(T_1+T_2) - 2\sin \mathcal{F}T_1 \cos \mathcal{F}(2T_1+3T_2) \\ - 2\sin \mathcal{F}T_2 \cos \mathcal{F}(T_1-2T_2) - \sin \mathcal{F}(T_1+T_2) \cos 2\mathcal{F}(T_1-T_2)\Big]\Bigg) \\ + \mathrm{Tr}_L\Bigg(\frac{\mathcal{F}^2 T_3}{\Delta_\mathcal{F} \sin \mathcal{F}T_2} \Big[\, \sin \mathcal{F}(T_1+T_2) \cos \mathcal{F}(2T_1+T_2) - \sin \mathcal{F}T_1 \cos 2\mathcal{F}(T_1+T_2)\Big] \end{aligned}$$

[1] In Eq. (4.19), one may replace $G^a_{\mu\nu}(\tau,\tau') \to G^a_{\mu\nu}(\tau,\tau') - G^a_{\mu\nu}(\tau,0) - G^a_{\mu\nu}(0,\tau')$ as seen in [11], however our final results do not change because we only need the derivatives $\partial_\tau \partial_{\tau'} \mathcal{G}^{ab}_{\mu\nu}$.

$$+ \frac{\mathcal{F}}{\Delta_{\mathcal{F}}}\Big[3\sin\mathcal{F}T_1\cos\mathcal{F}(2T_1+T_2)+\cos 2\mathcal{F}T_1\sin\mathcal{F}(T_1+T_2)\Big]\Bigg)\cdot \mathrm{Tr}_L\Bigg(\cos 2\mathcal{F}T_2\Bigg)$$

$$+\,\mathrm{Tr}_L\Bigg(\frac{\mathcal{F}^2T_3}{\Delta_{\mathcal{F}}\sin\mathcal{F}T_2}\Big[\sin\mathcal{F}T_2\cos\mathcal{F}(T_1-T_2)+\cos\mathcal{F}T_2\sin\mathcal{F}(T_1+T_2)$$

$$-\sin\mathcal{F}T_1\cos 2\mathcal{F}T_2\Big]+\frac{\mathcal{F}}{\Delta_{\mathcal{F}}}\sin\mathcal{F}T_1\cos\mathcal{F}T_2\Bigg)\cdot\mathrm{Tr}_L\Bigg(\cos 2\mathcal{F}(T_1+T_2)\Bigg)$$

$$+\,\delta(T_2)\,2(1-D)\,\mathrm{Tr}_L\Bigg(\cos 2\mathcal{F}T_1\Bigg)+\,\delta(T_3)\mathrm{Tr}_L\Bigg(\cos 2\mathcal{F}(T_1-T_2)\Bigg)$$

$$-\delta(T_3)\mathrm{Tr}_L\Bigg(\cos 2\mathcal{F}T_1\Bigg)\cdot\mathrm{Tr}_L\Bigg(\cos 2\mathcal{F}T_2\Bigg)\Bigg\}\int d^Dx_0\ , \tag{4.21}$$

where

$$\Delta_{\mathcal{F}}=\sin\mathcal{F}T_1\sin\mathcal{F}T_2+\mathcal{F}T_3\sin\mathcal{F}(T_1+T_2)\ . \tag{4.22}$$

4.2 The $I_2[A]$ part

The computation of the other quantity $I_2[A]$ is similar to the above calculations, however the topology of the world-line diagram is different in this case. Let us start with the following expression. First, Eq. (2.3) with (3.7) inserted becomes

$$\begin{aligned} I_2[\mathcal{A}] \;&=\; \frac{1}{4}\int_0^\infty dT_1dT_2\oint[\mathcal{D}x_1]_{T_1}\oint[\mathcal{D}x_2]_{T_2}\ \delta(x_1(0)-x_2(0)) \\ &\quad\times\mathrm{Tr}_C\Big[\lambda^a\exp\Big\{\mathcal{M}_1^{(+)}\otimes\mathcal{T}_-\Big\}_{\mu\nu}\Big]\mathrm{Tr}_C\Big[\lambda^a\exp\Big\{\mathcal{M}_2^{(+)}\otimes\mathcal{T}_-\Big\}_{\nu\mu}\Big]\ . \end{aligned} \tag{4.23}$$

With the expansion (4.4) and the properties

$$\mathrm{Tr}_C\,(\lambda^a\,\mathcal{T}_+)=\mathrm{Tr}_C\,(\lambda^a\,\mathcal{I})=0\ , \tag{4.24}$$

$$\mathrm{Tr}_C\,(\lambda^a\,\mathcal{T}_-)=n^b\,\mathrm{Tr}_C\left(\lambda^a\lambda^b\right)=n^b\,2\,\delta^{ab}=2\,n^a\ , \tag{4.25}$$

we have the formula

$$\mathrm{Tr}_C\Big[\lambda^a\exp\Big\{\mathcal{M}_k^{(\kappa)}\otimes\mathcal{T}_-\Big\}_{\rho\sigma}\Big]=2\,n^a\sinh\Big\{\mathcal{M}_k^{(\kappa)}\Big\}_{\rho\sigma}\ . \tag{4.26}$$

Remembering the relation $n^an^a=1$, we then derive from (4.23)

$$\begin{aligned} I_2[\mathcal{A}] \;&=\; \int_0^\infty dT_1dT_2\oint[\mathcal{D}x_1]_{T_1}\oint[\mathcal{D}x_2]_{T_2}\ \delta(x_1(0)-x_2(0))\,\mathrm{tr}_L\Big[\sinh\Big\{\mathcal{M}_1^{(+)}\Big\}\sinh\Big\{\mathcal{M}_2^{(+)}\Big\}\Big] \\ &=\; \frac{1}{4}\int_0^\infty dT_1dT_2\Big[\mathcal{N}^{(+,+)}\mathrm{Tr}_L(e^{2i(T_1+T_2)\mathcal{F}})\ -\mathcal{N}^{(+,-)}\mathrm{Tr}_L(e^{2i(T_1-T_2)\mathcal{F}})\Big] \\ &\quad+\quad(\mathcal{F}\to-\mathcal{F})\ , \end{aligned} \tag{4.27}$$

where

$$\mathcal{N}^{(\kappa_1,\kappa_2)} = \int d^D y_1 \int d^D y_2 \left[\prod_{k=1}^{2} \int\limits_{x_k(0)=y_1}^{x_k(T_k)=y_2} \mathcal{D}x_k\right] \delta^D(y_1 - y_2) e^{S(\kappa_1,\kappa_2)} \tag{4.28}$$

with

$$S^{(\kappa_1,\,\kappa_2)} = -\frac{1}{4}\sum_{k=1}^{2}\int\limits_0^{T_k} d\tau[\dot{x}_k^2 + 2i\,\kappa_k\, x_{k\sigma}\mathcal{F}_{\sigma\rho}\,\dot{x}_{k\rho}]\ . \tag{4.29}$$

The normalizations $\mathcal{N}^{(\kappa_1,\kappa_2)}$ satisfy the following properties:

$$\mathcal{N}^{(+,+)} = \mathcal{N}^{(+,-)} = \mathcal{N}^{(-,+)} = \mathcal{N}^{(-,-)}, \tag{4.30}$$

since, in the present case, the inversions of paths (i.e., the changes of κ's signs) do not change the value of the path integral (4.28): note that all the initial and ending points of two closed loops are identical. The $(\mathcal{F} \to -\mathcal{F})$ terms in (4.27) lead again to a factor of two, and thus we have

$$I_2[\mathcal{A}] = \frac{1}{2}\int\limits_0^{\infty} dT_1 dT_2\, \mathcal{N}^{(+,-)}\, \mathrm{Tr}_L(e^{2i(T_1+T_2)\mathcal{F}} - e^{2i(T_1-T_2)\mathcal{F}})\ . \tag{4.31}$$

The quantity (4.28) can be evaluated as follows. Recalling the relation

$$\delta^D(y_1 - y_2) = \lim_{T_3\to 0}\left(\frac{1}{4\pi T_3}\right)^{\frac{D}{2}} \exp\Big[-\frac{1}{4T_3}(y_1 - y_2)^2\Big]\ , \tag{4.32}$$

we convert the δ-function to the following path integral form:

$$\delta^D(y_1 - y_2) = \lim_{T_3\to 0}\int\limits_{x_3(0)=y_1}^{x_3(T_3)=y_2} \mathcal{D}x_3\, \exp\Big[-\frac{1}{4}\int\limits_0^{T_3} \dot{x}_3^2(\tau)d\tau\Big]\ , \tag{4.33}$$

and this leads to the relations

$$\mathcal{N}^{(\kappa_1,\kappa_2)} = \lim_{T_3\to 0}\mathcal{N}^{(\kappa_1,\kappa_2,0)} = (4\pi)^{-D}\mathrm{det}_L^{-1/2}\left(\frac{\sin\mathcal{F}T_1\sin\mathcal{F}T_2}{\mathcal{F}^2}\right)\int d^D x_0\ . \tag{4.34}$$

We therefore have the expression

$$\begin{aligned} I_2[\mathcal{A}] \;&=\; \frac{1}{2}(4\pi)^{-D}\int\limits_0^{\infty} dT_1 dT_2\, \mathrm{det}_L^{-1/2}\left(\frac{\sin\mathcal{F}T_1\sin\mathcal{F}T_2}{\mathcal{F}^2}\right) \\ &\times\Big[\,\mathrm{Tr}_L\cos 2\mathcal{F}(T_1+T_2) - \mathrm{Tr}_L\cos 2\mathcal{F}(T_1-T_2)\Big]\int d^D x_0\ . \end{aligned} \tag{4.35}$$

It is interesting that the normalization $\mathcal{N}^{(\kappa_1,\kappa_2)}$ can still be obtained as a singular limit (the $T_3 \to 0$ limit) from Eq. (4.18), although I_2 is not a singular part of I_1 (cf. (4.21)) [11].

5 The world-line moduli integrals

In this section, we study the divergence structure of the Euler-Heisenberg-type action derived in the previous sections. In the world-line formalism, divergences come out explicitly after performing the proper time integrals (the T_a integrals; $a = 1, 2, 3$ in the present case), and hence we have to examine how to perform these integrals before discussing renormalizations. However, generally speaking, multi-integrations are difficult to perform in an analytic way, and hence we here consider the simplest case corresponding to gluon two-point function parts.

Let us consider the Taylor expansions of $\Gamma[A]$ concerning $\mathcal{F}$ (omitting constant terms) in the same way as done in the one-loop case. Expanding I_1 and I_2 up to the second order of $\mathcal{F}$, we extract the following quantities from Eqs. (4.21) and (4.35):

$$
\begin{aligned}
I_1[\mathcal{A}] &= \frac{g_0^4\mu^{4\varepsilon}}{(4\pi)^{4-2\varepsilon}}\int d^Dx_0\,\mathcal{F}_{\mu\nu}\mathcal{F}_{\nu\mu}\Big\{\,30C_1+42C_2+48C_3+\frac{93}{2}C_4-2C_5-10C_6\\
&\quad -(17C_1+29C_2+\frac{57}{2}C_3+32C_4-\frac{17}{3}C_6)\,\varepsilon\\
&\quad -(2C_1+2C_2+5C_3+6C_4-\frac{2}{3}C_6)\,\varepsilon^2\,\Big\}+\cdots\ , \qquad (5.1)\\
I_2[\mathcal{A}] &= \frac{g_0^4\mu^{4\varepsilon}}{(4\pi)^{4-2\varepsilon}}\int d^Dx_0\,\mathcal{F}_{\mu\nu}\mathcal{F}_{\nu\mu}\,(-4)\,C_5+\cdots\ , \qquad (5.2)
\end{aligned}
$$

with

$$
\begin{aligned}
C_1 &= \int_0^\infty dT_1dT_2dT_3\,\Delta^{\varepsilon-4}\,T_1^4T_2\,e^{-m^2(T_1+T_2+T_3)}\\
C_2 &= \int_0^\infty dT_1dT_2dT_3\,\Delta^{\varepsilon-4}\,T_1^3T_2^2\,e^{-m^2(T_1+T_2+T_3)}\\
C_3 &= \int_0^\infty dT_1dT_2dT_3\,\Delta^{\varepsilon-4}\,T_1^3T_2T_3\,e^{-m^2(T_1+T_2+T_3)}\\
C_4 &= \int_0^\infty dT_1dT_2dT_3\,\Delta^{\varepsilon-4}\,T_1^2T_2^2T_3\,e^{-m^2(T_1+T_2+T_3)}\\
C_5 &= \int_0^\infty dT_1dT_2dT_3\,\Delta^{\varepsilon-4}\,T_1^3T_2^3\,\delta(T_3)\,e^{-m^2(T_1+T_2+T_3)}\\
C_6 &= \int_0^\infty dT_1dT_2dT_3\,\Delta^{\varepsilon-4}\,T_1^4T_2^2\,\delta(T_3)\,e^{-m^2(T_1+T_2+T_3)}\ ,
\end{aligned}
\qquad (5.3)
$$

where we have put $D = 4 - 2\varepsilon$ and $\Delta = T_1T_2 + T_2T_3 + T_3T_1$. Also, the damping mass factor $e^{-m^2(T_1+T_2+T_3)}$ is inserted in each C_i in the same way as in the one-loop calculation (Section 3). In the meantime, we shall introduce the notation $\varepsilon' = \{\varepsilon; \varepsilon > 1\}$ for C_1 and C_6, in order not to mix it up with the (usual) infinitesimal parameter $\varepsilon > 0$. This description is indispensable for the convergency of the C_1 and C_6 integrals, although we shall set $\varepsilon' = \varepsilon$ after all, expecting the analytic continuation (see also Section 8-1-2 in [16]). The ε' divergences are related to the divergences from the artificial mass term insertion, since C_1 and C_6 contain tadpole contributions, which vanish in

the $m \to 0$ limit (in the sense of dimensional regularization).

In C_5 and C_6, all the integrals are easy to perform, and hence we simply write down

$$
\begin{aligned}
C_5 &= (m^2)^{-2\varepsilon}\,\Gamma^2(\varepsilon)\ , &(5.4)\\
C_6 &= (m^2)^{-2\varepsilon}\,\Gamma(\varepsilon+1)\Gamma(\varepsilon'-1)\ . &(5.5)
\end{aligned}
$$

The rest of C_i are computed in detail in Appendix C, and we only show the results as follows:

$$
\begin{aligned}
C_1 &= (m^2)^{-2\varepsilon}\frac{\Gamma(2\varepsilon)}{3-\varepsilon}\mathrm{B}(\varepsilon'-1,\varepsilon+2)+(m^2)^{-2\varepsilon}\frac{1}{64}\Gamma(\varepsilon+3)\Gamma(\varepsilon-3)\\
&\quad\times\Big[\,4\mathrm{B}(2,\frac{1}{2})\ F(2,3+\varepsilon,\frac{5}{2},\frac{1}{4})-3\mathrm{B}(3,\frac{1}{2})\ F(3,3+\varepsilon,\frac{7}{2},\frac{1}{4})\Big]\\
&+\ (m^2)^{-2\varepsilon}4^{-\varepsilon}\frac{\Gamma(2\varepsilon+1)}{(3-\varepsilon)(\varepsilon-2)}\Big[\,\mathrm{B}(\varepsilon,\frac{1}{2})_3F_2(1,2\varepsilon+1,\varepsilon;\varepsilon-1,\varepsilon+\frac{1}{2};\frac{1}{4})\\
&\quad-\frac{3}{4}\mathrm{B}(\varepsilon+1,\frac{1}{2})_3F_2(1,2\varepsilon+1,\varepsilon+1;\varepsilon-1,\varepsilon+\frac{3}{2},\frac{1}{4})\Big]\ , &(5.6)\\
C_2 &= (m^2)^{-2\varepsilon}\frac{1}{64}\Gamma(3+\varepsilon)\Gamma(\varepsilon-3)\mathrm{B}(3,\frac{1}{2})\ F(3,3+\varepsilon;\frac{7}{2};\frac{1}{4})\\
&+\ (m^2)^{-2\varepsilon}4^{-\varepsilon}\frac{\Gamma(2\varepsilon)}{3-\varepsilon}\mathrm{B}(\varepsilon,\frac{1}{2})_3F_2(2\varepsilon,1,\varepsilon;\varepsilon-2,\varepsilon+\frac{1}{2};\frac{1}{4})\ , &(5.7)\\
C_3 &= -C_4+(m^2)^{-2\varepsilon}\frac{1}{16}\frac{\Gamma(4+\varepsilon)\Gamma(\varepsilon-2)}{(\varepsilon+2)(\varepsilon+3)}\mathrm{B}(2,\frac{1}{2})_3F_2(4-\varepsilon,2+\varepsilon,2;3-\varepsilon,\frac{5}{2};\frac{1}{4})\\
&+\ (m^2)^{-2\varepsilon}4^{-\varepsilon}\Gamma(2\varepsilon)\frac{\Gamma(2-\varepsilon)}{\Gamma(4-\varepsilon)}\mathrm{B}(\varepsilon,\frac{1}{2})_3F_2(2,,2\varepsilon,\varepsilon;\varepsilon-1,\varepsilon+\frac{1}{2};\frac{1}{4})\ , &(5.8)\\
C_4 &= (m^2)^{-2\varepsilon}\frac{1}{32}\frac{\Gamma(4+\varepsilon)\Gamma(\varepsilon-2)}{(\varepsilon+2)(\varepsilon+3)}\mathrm{B}(3,\frac{1}{2})_3F_2(4-\varepsilon,2+\varepsilon,3;3-\varepsilon,\frac{7}{2};\frac{1}{4})\\
&+\ (m^2)^{-2\varepsilon}\frac{1}{2}4^{-\varepsilon}\Gamma(2\varepsilon)\frac{\Gamma(2-\varepsilon)}{\Gamma(4-\varepsilon)}\mathrm{B}(\varepsilon+1,\frac{1}{2})_3F_2(2,2\varepsilon,\varepsilon+1;\varepsilon-1,\varepsilon+\frac{3}{2};\frac{1}{4})\ . &(5.9)
\end{aligned}
$$

We can rewrite these expressions in terms of the hypergeometric function $F \equiv {}_2F_1$ only, and the results and their derivations are presented in Appendix C (see (C.20), (C.21), (C.38) and (C.39)). Since we could not find any convenient transformation formula from ${}_3F_2$ to ${}_2F_1$ in the literature, we have established a transformation technique in Appendix C. (A more concise explanation can be found in Appendix D.)

Let us consider the expressions (C.20), (C.21), (C.38) and (C.39). We now perform the Taylor expansions of the hypergeometric functions around $\varepsilon = 0$, in order to see the divergence structures of the coefficients C_i. Here, we are only interested in $1/\varepsilon$ terms in the sense of the MS scheme, and the hypergeometric functions which contribute to the desired pole terms are only generated from the following expansion:

$$
F(\alpha,a\varepsilon;\gamma+b\varepsilon;\frac{1}{4}) = F(\alpha,0;\gamma;\frac{1}{4})
$$

$$+ a\varepsilon F^{(0,1,0)}(\alpha,0;\gamma;\frac{1}{4}) + \, b\varepsilon F^{(0,0,1)}(\alpha,0;\gamma;\frac{1}{4}) + \mathcal{O}(\varepsilon^2) \ , \tag{5.10}$$

where

$$F^{(n,m,l)}(\alpha,\beta;\gamma;z) \overset{\text{def.}}{=} \partial_\alpha^n \, \partial_\beta^m \, \partial_\gamma^l \, F(\alpha,\beta;\gamma;z) \ . \tag{5.11}$$

These differential coefficients (for $\gamma \neq 0$) are evaluated by

$$F(\alpha,0;\gamma;z) = 1 \ , \qquad F^{(0,0,1)}(\alpha,0;\gamma;z) = 0 \ , \tag{5.12}$$

$$F^{(0,1,0)}(\alpha,0;\gamma;z) = F^{(1,0,0)}(0,\alpha;\gamma;z) = \frac{\alpha}{\gamma} z \, {}_3F_2(1,1,\alpha+1;2,\gamma+1;z) \ . \tag{5.13}$$

Here we again encounter the generalized hypergeometric function ${}_3F_2$, however in the present case, it can be reduced to the ordinary hypergeometric function ${}_2F_1$ through the following formula (derived in Appendix D):

$${}_3F_2(1,\beta,n+1;2,\gamma;z) = \frac{1}{n}\sum_{k=1}^{n} F(k,\beta;\gamma;z), \qquad (n \geq 1, |z| < 1, \Re(\gamma) > 0). \tag{5.14}$$

Combining (5.13) and (5.14), we have

$$F^{(0,1,0)}(n,0;\gamma;z) = \frac{z}{\gamma}\sum_{k=1}^{n} F(k,1;\gamma+1;z) \ , \qquad (n \geq 1, |z| < 1, \Re(\gamma) > 0), \tag{5.15}$$

and thus

$$F(n,a\varepsilon;\gamma+b\varepsilon;z) = 1 + \varepsilon \frac{az}{\gamma}\sum_{k=1}^{n} F(k,1;\gamma+1;z) + \mathcal{O}(\varepsilon^2) \ , \qquad (n \geq 1, |z| < 1, \Re(\gamma) > 0). \tag{5.16}$$

Owing to this formula, all coefficients in front of $1/\varepsilon$ can be written in terms of ${}_2F_1$ and the gamma functions. After some algebra, we obtain

$$\begin{aligned}
&C_1' = -\frac{1}{6\varepsilon^2} + \Big(-\frac{5}{9} + \frac{\rho_m}{3}\Big)\frac{1}{\varepsilon} + \mathcal{O}(1) \ , \qquad C_2' = \frac{1}{6\varepsilon^2} + \Big(\frac{1}{18} - \frac{\rho_m}{3}\Big)\frac{1}{\varepsilon} + \mathcal{O}(1) \ , \\
&C_3' = \frac{1}{12\varepsilon^2} - \Big(\frac{1}{72} + \frac{\rho_m}{6}\Big)\frac{1}{\varepsilon} + \mathcal{O}(1) \ , \qquad C_4' = -\frac{1}{12\varepsilon} + \mathcal{O}(1) \ , \\
&C_5' = \frac{1}{\varepsilon^2} - \frac{2\rho_m}{\varepsilon} + \mathcal{O}(1) \ , \qquad C_6' = -\frac{1}{\varepsilon} + \mathcal{O}(1) \ ,
\end{aligned} \tag{5.17}$$

where the overall factor $(4\pi\mu^2)^{2\varepsilon}$ seen in (5.1) and (5.2) is absorbed in C_i; i.e.,

$$C_i' = (4\pi\mu^2)^{2\varepsilon} C_i \ , \tag{5.18}$$

and we have defined

$$\rho_m = \gamma_{\scriptscriptstyle E} + \ln\frac{m^2}{4\pi\mu^2} \ , \qquad \gamma_{\scriptscriptstyle E} = \text{Euler const.} \tag{5.19}$$

Therefore from Eqs. (5.1), (5.2) and (5.17), we obtain

$$I_1[\mathcal{A}] \;=\; \frac{4g_0^4}{(4\pi)^4}\left[\frac{4}{\varepsilon^2}+\left(-\frac{11}{2}-8\rho_m\right)\frac{1}{\varepsilon}\right]\int d^Dx_0\left(-\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu}\right) \quad + \quad \mathcal{O}(\varepsilon^0) \tag{5.20}$$

$$I_2[\mathcal{A}] \;=\; \frac{4g_0^4}{(4\pi)^4}\left[-\frac{4}{\varepsilon^2}+\frac{8\rho_m}{\varepsilon}\right]\int d^Dx_0\left(-\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu}\right) \quad + \quad \mathcal{O}(\varepsilon^0) \tag{5.21}$$

and due to Eq. (2.1) the renormalization part of the effective action (purely gluon parts) at the 2nd order in $\mathcal{F}$ in our regularization is found to be

$$\Gamma[\mathcal{A}] = \frac{4g_0^4}{(4\pi)^4}\left(-\frac{11}{2\varepsilon}\right)\int d^Dx_0\left(-\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu}\right) \quad + \quad \mathcal{O}(\varepsilon^0) \tag{5.22}$$

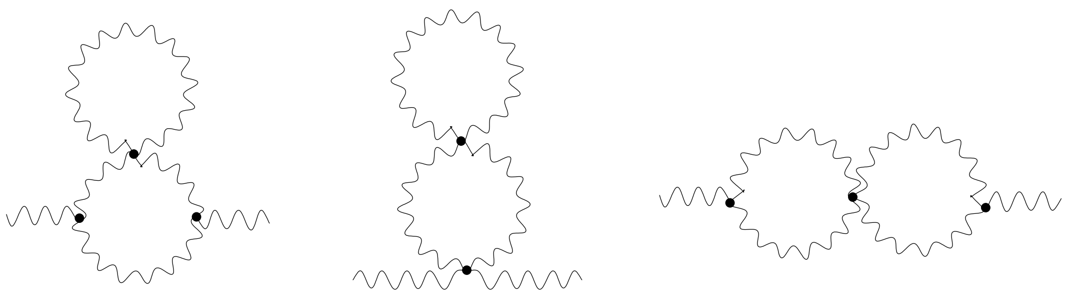


Figure 2: The “eight-figure” diagrams contained in the coefficients C_5 and C_6.

Finally, let us put a comment on what our results imply. Picking up C_5 and C_6 from Eqs. (5.1) and (5.2), and using (3.17), we extract the following quantities corresponding to self-energy parts:

$$\Pi_{5\,\mu\nu}^{\;ab} = g_0^4\frac{4\delta^{ab}}{(4\pi)^4}(k^2\delta_{\mu\nu}-k_\mu k_\nu)(-6C_5') \ , \tag{5.23}$$

$$\Pi_{6\,\mu\nu}^{\;ab} = g_0^4\frac{4\delta^{ab}}{(4\pi)^4}(k^2\delta_{\mu\nu}-k_\mu k_\nu)(-10C_6') \ . \tag{5.24}$$

As briefly shown in Appendix E, the Π_5 and Π_6 exactly coincide with the Feynman diagram results, if the coefficients of gluon kinetic terms are evaluated in the region close to the light cone $k^2 \to 0$; in other words, if k^2 is much smaller than the mass parameter m^2. (Note that $C_A = 2$ in the $su(2)$ case.) Thus, it is very natural to expect that the other coefficients C_1, C_2, C_3 and C_4 should possess the same meaning.

6 Conclusions and Discussions

In this paper, we have explicitly calculated the gluon parts of the two-loop Euler-Heisenberg actions, which are organized at the level of an implicit formulation in the previous paper [11]. The

present results are still preliminary to reach a clear physical quantity such as β-functions, however this paper is an important step toward a full-fledged extension of the world-line formalism to two-loop Yang-Mills theories. One of the main obstacles for this aim is the problem of how to integrate the proper time variables at higher loop calculations. Also, in the sense of field theory limit of string theory, this is an important problem: at the level of string theory, it is recognized as the problem how to perform the moduli integrals on a multiloop world-sheet.

We have performed the path integrals in Section 4, applying the world-line Green functions to all combinations of $su(2)$-like charges (the κ signatures; q.v. (4.11), (4.12), (4.14) and (4.15)). Then extracting the parts corresponding to the wave function renormalization, we have been able to perform the world-line "moduli" integrals to reveal the divergence structure in Section 5. In the standard method, it is difficult to perform the loop integrals containing a mass parameter, while in our case, all the integrations are carried out in Appendix C, giving rise to (generalized) hypergeometric functions as a result. This is certainly a significant point from a theoretical viewpoint, and hence we have mainly focused on the technical issue concerning the integrations. We should also note that the pseudo-abelian technique has worked out both at one- and two-loop levels, with reproducing the Feynman diagram results of Appendix E. We expect that our integration method will straightforwardly apply to higher order terms in $\mathcal{F}$ as well.

Although we have verified that our results contain correct Feynman diagram contributions, the followings should further be investigated as a next step toward our goal: the coincidence between the present results and those by the Feynman diagram method can only be understood in the region close to the light cone $k^2 \to 0$ ($k^2 << m^2$), where k_μ are the external gluon momenta. On the other hand, as seen in Section 3, we do not have the restriction on k^2 at the one-loop level, in order to extract the β-function. Similarly we shall encounter the same difference in the ghost loop calculations, and should clarify the reason for this kind of discrepancy. Related to this issue, another question is whether or not we can evaluate the pole structures of C_i for $m = 0$ (or $m^2 << k^2$). As inferred from Eqs. (E.14) and (E.16), the integrals C_i might depend on the region of either $k^2 << m^2$ or not. However the present calculations do not indicate such a dependence, simply because C_i are not the Fourier modes of gluon two-point function. To clarify this point, one should compute the correlator of two gluon vertex operators (of bosonic field representation) along the outline of Appendix B in Ref. [11]; in this case, the formulation should be extended to the super world-line formalism in order to optimize the inclusion of the four point interactions involving

external legs. Anyway, in order to find the correct β-function coefficient at two loops, we also have to add the contributions including the counter terms generated from one-loop divergences (in the massive formulation).

Following ref. [20] a gauge symmetry breaking IR gluon mass m^2 (introduced there only as a device to seperate IR and UV divergences) requires a further counter term

$$\frac{1}{2} Z_x m^2 A^a_\mu A^a_\mu \qquad \text{with} \qquad Z_x = -\frac{g^2 C_A}{16\pi^2 \varepsilon} \tag{6.1}$$

and $C_A = 2$ in our case, cancelling m^2 dependent singularities in the MS dimensional regularization scheme. Insertion of this counter term 'gluon mass' into the one-loop contribution of order $\mathcal{F}^2$ as found in Eq. (3.15), i.e. calculating with $m^2 \to m^2 + Z_x m^2$ and expanding to first order in $\frac{m^2}{\varepsilon}$, thus using

$$e^{-m^2 T} \to e^{-m^2 T} \left(1 - \frac{g^2 m^2 C_A}{16\pi^2 \varepsilon} T \right) , \tag{6.2}$$

yields, e.g. a singular two-loop $\mathcal{F}^2$-contribution:

$$\begin{aligned} \Gamma_{ct} &= \frac{C_A}{2(4\pi)^{D/2}} \Big(\frac{D}{12} - 2 \Big) \int_0^\infty dT\, T^{1-D/2}\, e^{-m^2 T} \Big(\frac{-g^2 m^2 C_A}{16\pi^2 \varepsilon} T \Big) \int d^D x_0 \, \mathrm{Tr}_L \mathcal{F}^2 & (6.3) \\ &= \frac{g^4 C_A^2}{(4\pi)^4} \left(\frac{10}{3\varepsilon} \right) \int d^D x_0 \left(-\frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}_{\mu\nu} \right) + \mathcal{O}(\varepsilon^0) \ . & (6.4) \end{aligned}$$

However in the background formalism there should be further counter terms including quantum as well as background fields. This has to be further analyzed in order to reproduce the usual β-function coefficient.

This paper concerns a theoretical interest, and is an important part of the ongoing long-term effort to find a way to getting over difficulties in the current calculation methods for higher loop amplitudes in field theory. Since the whole computation process in the world-line formalism looks completely different from the standard field theory calculations, the above questions are the milestones in the future study, and should be solved in order to make future practical applications successful.

Acknowledgement

We would like to thank M. Jamin for helpful discussions concerning infrared regularization.

A List of path reversal formulae

This appendix is a brief note on the reversals of path ordering and proper time directions. The standard definition of the path ordering (the normal type) is

$$\mathrm{P}\exp\left\{\int_{\tau_\alpha}^{\tau_\beta} d\tau M_\tau[x]\right\} = \sum_{n=0}^{\infty}\int_{\tau_\alpha}^{\tau_\beta} d\tau_1 \int_{\tau_\alpha}^{\tau_1} d\tau_2 \cdots \int_{\tau_\alpha}^{\tau_{n-1}} d\tau_n M_{\tau_1}[x]\cdots M_{\tau_n}[x] \ , \tag{A.1}$$

and the anti-path ordering (used in [11] for a certain reason) is

$$\mathrm{P}^*\exp\left\{\int_{\tau_\alpha}^{\tau_\beta} d\tau M_\tau[x]\right\} = \sum_{n=0}^{\infty}\int_{\tau_\alpha}^{\tau_\beta} d\tau_1 \int_{\tau_\alpha}^{\tau_1} d\tau_2 \cdots \int_{\tau_\alpha}^{\tau_{n-1}} d\tau_n M_{\tau_n}[x]\cdots M_{\tau_1}[x] \ . \tag{A.2}$$

We here assume that the M_τ in the above two definitions are the same objects.

The relations between the path and the anti-path ordering formulae are given by

$$W^{ae}_{\mu\nu}[x;T,0] = \mathrm{P}\exp\left\{\int_0^T d\tau M_\tau[x]\right\}^{ae}_{\mu\nu} = \mathrm{P}^*\exp\left\{\int_0^T d\tau M^T_\tau[x]\right\}^{ea}_{\nu\mu} \ , \tag{A.3}$$

$$W^{ae}_{\mu\sigma\rho\nu}[x;S,\tau_\alpha,0] = \mathrm{Tr}_C\left[\lambda^a\,\mathrm{P}^*\exp\left\{\int_0^{\tau_\alpha} d\tau M^T_\tau[x]\right\}_{\nu\rho}\lambda^e\,\mathrm{P}^*\exp\left\{\int_{\tau_\alpha}^{S} d\tau M^T_\tau[x]\right\}_{\sigma\mu}\right] , \tag{A.4}$$

where the M^T represents the transposition on both the color and Lorentz spaces.

Note that the transposition makes an additional minus sign in front of the $A\dot{x}$ term in M_τ. If we change the sign of gauge coupling g, this additional sign drops out, and the starting formulae presented in Sect. 2 follows from the previous paper directly.

The other useful observation is related to path inversion: As can be seen from Eq. (2.7) by distinguishing τ and τ' carefully (see Eq. (4.1)), the following relation holds:

$$\Big(M_\tau[x^{-1}]\Big)^{ab}_{\mu\nu} = \Big(M_{\tau_\alpha+\tau_\beta-\tau}[x]\Big)^{ba}_{\nu\mu} \ , \tag{A.5}$$

where x^{-1} denotes the inverted path x, each of them defined by

$$x : [\tau_\alpha, \tau_\beta] \to R^4, \qquad \tau \mapsto x(\tau) \ , \tag{A.6}$$

$$x^{-1} : [\tau_\alpha, \tau_\beta] \to R^4, \qquad \tau \mapsto x^{-1}(\tau) := x(\tau_\alpha+\tau_\beta-\tau) \tag{A.7}$$

respectively. As a consequence of Eq. (A.5) we have

$$\mathrm{P}\exp\left\{\int_{\tau_\alpha}^{\tau_\beta} d\tau M_\tau[x]\right\}^T = \mathrm{P}\exp\left\{\int_{\tau_\alpha}^{\tau_\beta} d\tau M_\tau[x^{-1}]\right\} \ , \tag{A.8}$$

where again T means transposition in both color as well as Lorentz space.

Describing path inversion by the x^{-1} symbol is a useful tool in performing world-line calculations (rather than changing the direction of τ or other possibilities), because it nicely fits together with the following identities for world-line path integrals[2]:

$$\int\limits_{x(0)=y_1}^{x(S)=y_2} \mathcal{D}x\ F[x] \quad = \int\limits_{x(0)=y_2}^{x(S)=y_1} \mathcal{D}x\ F[x^{-1}] \tag{A.9}$$

and consequently

$$\oint \mathcal{D}x\ F[x] = \int d^D y \int\limits_{x(0)=y}^{x(S)=y} \mathcal{D}x\ F[x] = \int d^D y \int\limits_{x(0)=y}^{x(S)=y} \mathcal{D}x\ F[x^{-1}] = \oint \mathcal{D}x\ F[x^{-1}]\ , \tag{A.10}$$

where F is an arbitrary functional and the integrands $F[x^{-1}]$ are to be understood as follows: for any path x integrated over within the path integral the corresponding path x^{-1} is to be constructed (in thoughts) and the functional is to be evaluated for this path x^{-1}. Thus, in the path integral, x^{-1} depends on x. Note that we also have

$$\begin{aligned}
\int\limits_{x(0)=y_1}^{x(S)=y_2} [\mathcal{D}x]_S\ F[x] \quad &= \int\limits_{x(0)=y_1}^{x(S)=y_2} \mathcal{D}x\ e^{-\int\limits_0^S d\tau\ \frac{1}{4}\dot{x}^2} F[x] = \int\limits_{x(0)=y_2}^{x(S)=y_1} \mathcal{D}x\ e^{-\int\limits_0^S d\tau\ \frac{1}{4}(\dot{x}^{-1})^2} F[x^{-1}] \\
&= \int\limits_{x(0)=y_2}^{x(S)=y_1} \mathcal{D}x\ e^{-\int\limits_0^S d\tau\ \frac{1}{4}\dot{x}^2} F[x^{-1}] = \int\limits_{x(0)=y_2}^{x(S)=y_1} [\mathcal{D}x]_S\ F[x^{-1}]\ ,
\end{aligned} \tag{A.11}$$

i.e. our usual bracket notation for the free path integral part is not affected.

Using these relations and identities one can easily verify that the gluon propagator in a backgound, given by the world-line representation [15]

$$\Delta^{ab}_{\mu\nu}(x_1, x_2) = \int\limits_0^\infty dT \int\limits_{x(0)=x_2}^{x(T)=x_1} [\mathcal{D}x]_T\ \mathrm{P}\exp\left\{ \int\limits_0^T d\tau M_\tau[x] \right\}^{ab}_{\mu\nu}\ , \tag{A.12}$$

satisfies the following property:

$$\Delta^{ab}_{\mu\nu}(x_1, x_2) = \Delta^{ba}_{\nu\mu}(x_2, x_1)\ . \tag{A.13}$$

[2]These identities can easily be verified from the path integral discretization.

B The derivation of $I_1[\mathcal{A}]$

In this Appendix, we show some details for the computation in Section 4.1. First, we need various values of the quantity (4.17) at $(\tau,\tau') = (0,T_3)$ for $a,b = 1,3$, and those are given by the following. The necessary derivatives of the Green functions are

$$\partial_\tau\partial_{\tau'}\mathcal{G}^{13}_{\mu\nu}(\tau,\tau';+,0,-)\Big|_{\tau=0,\tau'=T_3} = \left(\frac{2T_2\mathcal{F}^2\, e^{i\mathcal{F}(T_3-T_1)}}{\sin\mathcal{F}T_1\sin\mathcal{F}T_3+\mathcal{F}T_2\sin\mathcal{F}(T_1+T_3)}\right)_{\mu\nu} \tag{B.1}$$

$$\partial_\tau\partial_{\tau'}\mathcal{G}^{13}_{\mu\nu}(\tau,\tau';0,-,+)\Big|_{\tau=0,\tau'=T_3} = \left(\frac{2\mathcal{F}\sin\mathcal{F}T_2\, e^{-i\mathcal{F}T_3}}{\sin\mathcal{F}T_2\sin\mathcal{F}T_3+\mathcal{F}T_1\sin\mathcal{F}(T_2+T_3)}\right)_{\mu\nu} \tag{B.2}$$

$$\partial_\tau\partial_{\tau'}\mathcal{G}^{13}_{\mu\nu}(\tau,\tau';+,-,0)\Big|_{\tau=0,\tau'=T_3} = \left(\frac{2\mathcal{F}\sin\mathcal{F}T_2\, e^{-i\mathcal{F}T_1}}{\sin\mathcal{F}T_1\sin\mathcal{F}T_2\,+\mathcal{F}T_3\sin\mathcal{F}(T_1+T_2)}\right)_{\mu\nu} \tag{B.3}$$

$$\begin{aligned}\partial_\tau\partial_{\tau'}\mathcal{G}^{33}_{\mu\nu}(\tau,\tau';+,0,-)\Big|_{\tau=0,\tau'=T_3} &= \left(\frac{2T_2\mathcal{F}^2\, e^{i2\mathcal{F}T_3}\sin\mathcal{F}T_1\,\mathrm{cosec}\,\mathcal{F}T_3}{\sin\mathcal{F}T_1\sin\mathcal{F}T_3+\mathcal{F}T_2\sin\mathcal{F}(T_1+T_3)}\right)_{\mu\nu}\\ &\quad+2\left(\mathbf{1}_L\delta(T_3)-\frac{\mathcal{F}}{\sin\mathcal{F}T_3}e^{i\mathcal{F}T_3}\right)_{\mu\nu}\end{aligned} \tag{B.4}$$

$$\begin{aligned}\partial_\tau\partial_{\tau'}\mathcal{G}^{33}_{\mu\nu}(\tau,\tau';0,-,+)\Big|_{\tau=0,\tau'=T_3} &= \left(\frac{2T_1\mathcal{F}^2\, e^{-i2\mathcal{F}T_3}\sin\mathcal{F}T_2\,\mathrm{cosec}\,\mathcal{F}T_3}{\sin\mathcal{F}T_2\sin\mathcal{F}T_3+\mathcal{F}T_1\sin\mathcal{F}(T_2+T_3)}\right)_{\mu\nu}\\ &\quad+2\left(\mathbf{1}_L\delta(T_3)-\frac{\mathcal{F}}{\sin\mathcal{F}T_3}e^{-i\mathcal{F}T_3}\right)_{\mu\nu}\end{aligned} \tag{B.5}$$

$$\begin{aligned}\partial_\tau\partial_{\tau'}\mathcal{G}^{33}_{\mu\nu}(\tau,\tau';+,-,0)\Big|_{\tau=0,\tau'=T_3} &= \left(\frac{2T_3^{-1}\sin\mathcal{F}T_1\sin\mathcal{F}T_2}{\sin\mathcal{F}T_1\sin\mathcal{F}T_2+\mathcal{F}T_3\sin\mathcal{F}(T_1+T_2)}\right)_{\mu\nu}\\ &\quad+2(\delta(T_3)-\frac{1}{T_3})(\mathbf{1}_L)_{\mu\nu}\ .\end{aligned} \tag{B.6}$$

The normalization constants are given by

$$\mathcal{N}^{(+,-,0)} = (4\pi)^{-D}\det{}_L^{1/2}\left(\frac{\mathcal{F}^2}{\sin\mathcal{F}T_1\sin\mathcal{F}T_2+\mathcal{F}T_3\sin\mathcal{F}(T_1+T_2)}\right)\int d^Dx_0\ , \tag{B.7}$$

and the other values can be obtained by exchanging the labels a on κ_a and T_a simultaneously; for example,

$$\mathcal{N}^{(+,0,-)} = \mathcal{N}^{(+,-,0)}\Big|_{T_2\leftrightarrow T_3}. \tag{B.8}$$

Note also

$$\mathcal{N}^{(+,+,0)} = \mathcal{N}^{(-,-,0)} = \mathcal{N}^{(\pm,\mp,0)}\ . \tag{B.9}$$

Now, plugging in (B.1–B.6) and the corresponding normalizations, for each line of Eqs. (4.14) and (4.15) we get an expression of the following structure:

$$\mathcal{N}^{(\kappa_1,\kappa_2,\kappa_3)}\ [\mathrm{Tr}_L(\text{power series in }\mathcal{F})+\mathrm{Tr}_L(\text{power series in }\mathcal{F})\cdot\mathrm{Tr}_L(\text{power series in }\mathcal{F})]\ . \tag{B.10}$$

Because of the antisymmetry of $\mathcal{F}$, only even powers of $\mathcal{F}$ contribute to the traces. Furthermore we have the property

$$\mathcal{N}^{(\kappa_1,\kappa_2,\kappa_3)}(\mathcal{F}) = \mathcal{N}^{(\kappa_1,\kappa_2,\kappa_3)}(-\mathcal{F}) \ , \tag{B.11}$$

and thus (B.10) is invariant with respect to $\mathcal{F} \to -\mathcal{F}$. Therefore the $\mathcal{F} \to -\mathcal{F}$ terms in Eqs. (4.14) and (4.15) just give a factor of two.

Using this fact, the expressions (B.1–B.6) and the corresponding normalizations we obtain from Eqs. (4.14) and (4.15) straightforwardly:

$$\begin{aligned}
\Gamma_2[\mathcal{A}] = -\frac{1}{2}\,(4\pi)^{-D} \int_0^\infty & dT_1 dT_2 dT_3 \,\det{}_L^{1/2}\left(\frac{\mathcal{F}^2}{\Delta_{\mathcal{F}}^{(2)}}\right)\Bigg\{ \\
& \mathrm{Tr}_L\Bigg(\frac{\mathcal{F}^2 T_2}{\Delta_{\mathcal{F}}^{(2)}}\Big[-2\cos\mathcal{F}(T_1-T_3)\cos 2\mathcal{F}(T_1+T_3) + \cos\mathcal{F}(T_3-T_1) \\
& + \cos\mathcal{F}(T_1-T_3)\mathrm{Tr}_L \cos 2\mathcal{F}(T_1+T_3)\Big]\Bigg) \\
& + 2\mathrm{Tr}_L\Bigg(\frac{\mathcal{F}\sin\mathcal{F}T_1}{\Delta_{\mathcal{F}}^{(2)}}\Big[2\sin\mathcal{F}T_3 \sin 2\mathcal{F}(T_1+T_3) - \cos\mathcal{F}(2T_1-T_3) \\
& + \cos\mathcal{F}(2T_1+T_3)\mathrm{Tr}_L \cos 2\mathcal{F}T_3\Big]\Bigg)\Bigg\}\int d^D x_0 \ ,
\end{aligned} \tag{B.12}$$

and

$$\begin{aligned}
\Gamma_1[\mathcal{A}] = -\frac{1}{2}\,(4\pi)^{-D} \int_0^\infty & dT_1 dT_2 dT_3 \,\det{}_L^{1/2}\left(\frac{\mathcal{F}^2}{\Delta_{\mathcal{F}}^{(2)}}\right)\Bigg\{ \\
\mathrm{Tr}_L\Bigg(& \frac{\mathcal{F}}{\sin\mathcal{F}T_3}\Big[-2\cos\mathcal{F}(2T_1+3T_3) + \cos\mathcal{F}(2T_1+T_3)\mathrm{Tr}_L\cos 2\mathcal{F}T_3 \\
& + 2\cos\mathcal{F}T_3 \mathrm{Tr}_L \cos 2\mathcal{F}(T_1+T_3)\Big]\Bigg) \\
& + \mathrm{Tr}_L\Bigg(\Big(\frac{\sin\mathcal{F}T_1\sin\mathcal{F}T_3}{\Delta_{\mathcal{F}}^{(2)}T_2} - \frac{1}{T_2}\Big)\Big[\cos 2\mathcal{F}(T_3-T_1) - \cos 2\mathcal{F}T_3 \mathrm{Tr}_L \cos 2\mathcal{F}T_1\Big]\Bigg) \\
& + \mathrm{Tr}_L\Bigg(\frac{\mathcal{F}^2 T_2 \sin\mathcal{F}T_1}{\Delta_{\mathcal{F}}^{(2)}\sin\mathcal{F}T_3}\Big[2\cos 2\mathcal{F}(T_1+2T_3) - \cos 2\mathcal{F}(T_1+T_3)\mathrm{Tr}_L\cos 2\mathcal{F}T_3 \\
& - \cos 2\mathcal{F}T_3 \mathrm{Tr}_L \cos 2\mathcal{F}(T_1+T_3)\Big]\Bigg) + \delta(T_3)\,2(1-D)\,\mathrm{Tr}_L\cos 2\mathcal{F}T_1 \\
& + \delta(T_2)\Big[\mathrm{Tr}_L\cos 2\mathcal{F}(T_1-T_3) - \mathrm{Tr}_L(\cos 2\mathcal{F}T_1)\cdot\mathrm{Tr}_L(\cos 2\mathcal{F}T_3)\Big]\Bigg\}\int d^D x_0 \ ,
\end{aligned} \tag{B.13}$$

with

$$\Delta_{\mathcal{F}}^{(2)} = \sin\mathcal{F}T_1 \sin\mathcal{F}T_3 + \mathcal{F}T_2 \sin\mathcal{F}(T_1+T_3) \ . \tag{B.14}$$

The sum of (B.12) and (B.13) gives $I_1[\mathcal{A}]$ by definition, and is certainly equivalent to Eq. (4.21). In order to obtain Eq. (4.21) itself, we should further take the following modification into account.

Exchanging T_2 and T_3, and using the relations followed from (B.14)

$$\frac{\mathcal{F}}{\sin \mathcal{F}T_3} = \frac{\mathcal{F}}{\Delta_{\mathcal{F}}^{(2)}} \sin \mathcal{F}T_1 + \frac{\mathcal{F}^2 T_2 \sin \mathcal{F}(T_1+T_3)}{\Delta_{\mathcal{F}}^{(2)} \sin \mathcal{F}T_3} \ , \tag{B.15}$$

$$\frac{1}{T_2} - \frac{\sin \mathcal{F}T_1 \sin \mathcal{F}T_3}{\Delta_{\mathcal{F}}^{(2)} T_2} = \frac{\mathcal{F}}{\Delta_{\mathcal{F}}^{(2)}} \sin \mathcal{F}(T_1+T_3) \ , \tag{B.16}$$

we finally arrive at the full expression shown in Eq. (4.21).

C Computational details of C_i

In this Appendix, we show the details of how to perform all the integrals in C_1, C_2, C_3 and C_4. Let us first perform the T_3 integrals in C_i. Applying the following formula to the T_3 parts in (5.3)

$$\int_0^\infty e^{-pt}(1+at)^{-\nu} dt = p^{\nu-1} a^{-\nu} e^{p/a}\, \Gamma(1-\nu; p/a) \ , \qquad (a>0) \tag{C.1}$$

and then transforming the T_1 and T_2 integrals with

$$\int_0^\infty dT_1 dT_2\, f(T_1,T_2) = \int_0^\infty dT T \int_0^1 du f(T(1-u), Tu) \ , \tag{C.2}$$

we obtain the followings for $i=1$ and 2 (further using $m^2 T = t$)

$$C_i = (m^2)^{-2\varepsilon} \int_0^\infty dt \int_0^1 du f_i(u) t^{2+\varepsilon} e^{-t+tu(1-u)} \Gamma(\varepsilon-3; tu(1-u)), \qquad (i=1,2) \tag{C.3}$$

and for $i=3$ and 4

$$\begin{aligned} C_i \ &= \ -(m^2)^{-2\varepsilon} \int_0^\infty dt \int_0^1 du f_i(u) t^{\varepsilon} e^{-t+tu(1-u)} \Big[(3-\varepsilon) t \Gamma(\varepsilon-3; tu(1-u)) \\ &\quad + t^2 u(1-u) \Gamma(\varepsilon-3; tu(1-u)) + t^2 u(1-u) \frac{\partial}{\partial \xi} \Gamma(\varepsilon-3; \xi) \Big] \ , \end{aligned} \tag{C.4}$$

where we have defined

$$\xi = tu(1-u) \tag{C.5}$$

and

$$f_i(u) = u^{i-2a}(1-u)^{a+5-i}, \qquad a = \Big[\frac{i-1}{2}\Big]_G \tag{C.6}$$

with Gauss' integer symbol $[\ \ ]_G$. In the following, we evaluate (C.3) and (C.4) separately because we shall proceed on different technique and formulae.

C.1 C_1 and C_2

We now consider the t integration in (C.3). In the first place, it can be integrated in terms of the formula (6.455.1 in [19]):

$$\int_0^\infty t^{\mu-1}e^{-pt}\Gamma(\nu,\alpha t)\,dt = \frac{\alpha^\nu\Gamma(\mu+\nu)}{\mu(\alpha+p)^{\mu+\nu}}\ F(1,\mu+\nu;\mu+1;\frac{p}{\alpha+p}),$$
$$\mathrm{Re}\,(\alpha+p), \mathrm{Re}\,\mu, \mathrm{Re}\,(\mu+\nu)>0\ . \tag{C.7}$$

Then applying the formula (9.131.2 in [19]),

$$\begin{aligned} F(\alpha,\beta;\gamma;z) &= \frac{\Gamma(\alpha+\beta-\gamma)\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\beta)}(1-z)^{\gamma-\alpha-\beta}\ F(\gamma-\alpha,\gamma-\beta;\gamma-\alpha-\beta+1;1-z) \\ &+ \frac{\Gamma(\gamma)\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}\ F(\alpha,\beta;\alpha+\beta-\gamma+1;1-z), \end{aligned} \tag{C.8}$$

we have

$$\begin{aligned} C_i &= (m^2)^{-2\varepsilon}\int_0^1 du f_i(u)\Gamma(\varepsilon+3)\Gamma(\varepsilon-3)F(4-\varepsilon,3+\varepsilon;4-\varepsilon;u(1-u)) \\ &+ (m^2)^{-2\varepsilon}\int_0^1 du f_i(u)\Big[u(1-u)\Big]^{\varepsilon-3}\frac{\Gamma(2\varepsilon)}{3-\varepsilon}F(2\varepsilon,1;\varepsilon-2;u(1-u))\ . \end{aligned} \tag{C.9}$$

In order to perform the u integrations, we consider the following change of variable: cutting the integration region $(0,1)$ in half, define u_+ (u_-) for larger (smaller) values of u, and apply

$$u_\pm = \frac{1}{2}(1\pm\sqrt{1-y})\ . \tag{C.10}$$

With this change of variable, the following formula holds for an arbitrary function $H(u)$

$$\int_0^1 du f_i(u)H(u(1-u)) = \frac{1}{4}\int_0^1 \frac{dy}{\sqrt{1-y}}g_i(y)H(y/4)\ , \tag{C.11}$$

where we have defined $g_i(y)$ for all i:

$$g_i(y) \equiv f_i\Big(\frac{1+\sqrt{1-y}}{2}\Big) + f_i\Big(\frac{1-\sqrt{1-y}}{2}\Big)\ , \tag{C.12}$$

and we hence have

$$g_1(y) = \frac{1}{16}y(4-3y), \qquad g_2(y) = \frac{1}{16}y^2, \tag{C.13}$$
$$g_3(y) = \frac{1}{8}y(2-y), \qquad g_4(y) = \frac{1}{8}y^2. \tag{C.14}$$

At a glance, one may realize that the y integration can be performed by the formula (7.512.12 [19])

$$\int_0^1 (1-t)^{\mu-1}t^{\nu-1}F(\alpha,\beta;\gamma;zt)\,dt = \mathrm{B}(\mu,\nu)\,{}_3F_2(\alpha,\beta,\nu;\gamma,\mu+\nu;z),$$
$$(\mathrm{Re}\,\mu, \mathrm{Re}\,\nu>0, |z|<1) \tag{C.15}$$

and this is exactly the way we obtain the generalized hypergeometric ${}_3F_2$ expressions (5.6) and (5.7).

In order to further derive hypergeometric ${}_2F_1$ expressions, we rather notice the special case $\gamma = \nu$ in (C.15). Before applying (C.15), we adjust auxiliary variables of ${}_2F_1$ until the special case is applicable, with using the formula (9.137.18 [19])

$$F(\alpha,\beta;\gamma;z) = \frac{\gamma-\alpha}{\gamma} F(\alpha,\beta;\gamma+1;z) + \frac{\alpha}{\gamma} F(\alpha+1,\beta;\gamma+1;z) \ . \tag{C.16}$$

Before writing down the final results for C_1 and C_2, we here remark on the way how C_1 contains the ε' dependence. Using the following formula (9.137.11 [19]) with $\beta = 0$

$$\gamma F(\alpha,\beta;\gamma;z) - \gamma F(\alpha,\beta+1;\gamma;z) + \alpha z F(\alpha+1,\beta+1;\gamma+1;z) = 0 \ , \tag{C.17}$$

and then decomposing the second term in (C.9)

$$F(2\varepsilon,1;\varepsilon-2;\frac{y}{4}) = 1 + \frac{2\varepsilon}{\varepsilon-2}\,\frac{y}{4}\ F(2\varepsilon+1,1;\varepsilon-1;\frac{y}{4}) \ , \tag{C.18}$$

we extract the following integral from C_1:

$$\begin{aligned} l_1 \ &\overset{\text{def.}}{=}\ 4^{-\varepsilon}\Gamma(2\varepsilon)\int_0^1 y^{\varepsilon-2}(4-3y)(1-y)^{-1/2}\,dy \\ &=\ \Gamma(2\varepsilon)\int_0^1 u^{\varepsilon-2}(1-u)^{\varepsilon+1}\,du \ =\ \Gamma(2\varepsilon)\mathrm{B}(\varepsilon'-1,\varepsilon+2) \ . \end{aligned} \tag{C.19}$$

Evaluating the l_1 contribution separately, we obtain the results written in terms of ${}_2F_1$ only:

$$\begin{aligned} C_1 \ =\ & (m^2)^{-2\varepsilon}\frac{\Gamma(2\varepsilon)}{3-\varepsilon}\mathrm{B}(\varepsilon'-1,\varepsilon+2) + (m^2)^{-2\varepsilon}\frac{1}{64}\Gamma(\varepsilon+3)\Gamma(\varepsilon-3) \\ & \times\Big[\,4\mathrm{B}(2,\frac{1}{2})\ F(2,3+\varepsilon;\frac{5}{2};\frac{1}{4}) - 3\mathrm{B}(3,\frac{1}{2})\ F(3,3+\varepsilon;\frac{7}{2};\frac{1}{4})\Big] \\ + \ & (m^2)^{-2\varepsilon}4^{-e}\frac{\Gamma(2\varepsilon+1)\mathrm{B}(\varepsilon,\frac{1}{2})}{(3-\varepsilon)(\varepsilon-2)(\varepsilon-1)} \\ & \times\Big[\,(\varepsilon-2)F(1,2\varepsilon+1;\varepsilon+\frac{1}{2};\frac{1}{4}) + F(2,2\varepsilon+1;\varepsilon+\frac{1}{2};\frac{1}{4})\Big] \\ - \ & (m^2)^{-2\varepsilon}4^{-e}\frac{3}{4}\frac{\Gamma(2\varepsilon+1)\mathrm{B}(\varepsilon+1,\frac{1}{2})}{(3-\varepsilon)(\varepsilon-2)}\Big[\,\frac{\varepsilon-2}{\varepsilon}F(1,2\varepsilon+1;\varepsilon+\frac{3}{2};\frac{1}{4}) \\ & + \frac{2(\varepsilon-2)}{\varepsilon(\varepsilon-1)}\ F(2,2\varepsilon+1;\varepsilon+\frac{3}{2};\frac{1}{4}) + \frac{2}{\varepsilon(\varepsilon-1)}F(3,2\varepsilon+1;\varepsilon+\frac{3}{2};\frac{1}{4})\Big], \end{aligned} \tag{C.20}$$

$$\begin{aligned} C_2 \ =\ & (m^2)^{-2\varepsilon}\frac{1}{64}\Gamma(3+\varepsilon)\Gamma(\varepsilon-3)\mathrm{B}(3,\frac{1}{2})F(3,3+\varepsilon;\frac{7}{2};\frac{1}{4}) \\ + \ & (m^2)^{-2\varepsilon}4^{-\varepsilon}\frac{\Gamma(2\varepsilon)\mathrm{B}(\varepsilon,\frac{1}{2})}{1-\varepsilon}\Big[\ F(1,2\varepsilon;\varepsilon+\frac{1}{2};\frac{1}{4}) \\ & -\frac{2}{2-\varepsilon}F(2,2\varepsilon;\varepsilon+\frac{1}{2};\frac{1}{4}) + \frac{2}{(2-\varepsilon)(3-\varepsilon)}\ F(3,2\varepsilon;\varepsilon+\frac{1}{2};\frac{1}{4})\Big] \ . \end{aligned} \tag{C.21}$$

C.2 C_3 and C_4

In this case, we split C_i; $i=3,4$ as

$$C_i = -(m^2)^{-2\varepsilon}\Big[(3-\varepsilon)R_1 + R_2 + R_3\Big], \tag{C.22}$$

where

$$\begin{aligned} R_1 &= \int_0^1 du\, f_i(u) \int_0^\infty t^{1+\varepsilon} e^{-t+tu(1-u)} \Gamma(\varepsilon-3; tu(1-u))\, dt \ , & \text{(C.23)} \\ R_2 &= \int_0^1 du\, f_i(u) u(1-u) \int_0^\infty t^{2+\varepsilon} e^{-t+tu(1-u)} \Gamma(\varepsilon-3; tu(1-u))\, dt \ , & \text{(C.24)} \\ R_3 &= \int_0^1 du\, f_i(u) u(1-u) \int_0^\infty t^{2+\varepsilon} e^{-t+tu(1-u)} \frac{\partial}{\partial\xi}\Gamma(\varepsilon-3;\xi)\, dt \ . & \text{(C.25)} \end{aligned}$$

First, we integrate R_2 using (C.7) in the same way as done in (C.3):

$$R_2 = \frac{\Gamma(2\varepsilon)}{\varepsilon+3}\int_0^1 f_i(u)\Big[u(1-u)\Big]^{\varepsilon-2} F(1,2\varepsilon;\varepsilon+4;1-u(1-u))\, du. \tag{C.26}$$

Second, we notice that the incomplete gamma function is related to the Whittaker function:

$$\Gamma(\nu,z) = z^{\frac{\nu-1}{2}} e^{-z/2} W_{\frac{\nu-1}{2},\frac{\nu}{2}}(z), \tag{C.27}$$

and $W_{\kappa,\nu}(z)$ satisfy

$$\begin{aligned} & W_{\kappa,\mu}(z) = W_{\kappa,-\mu}(z), & \text{(C.28)} \\ & z\partial_z W_{\kappa,\mu}(z) = \Big(\frac{z}{2}-\kappa\Big) W_{\kappa,\mu}(z) - W_{\kappa+1,\mu}(z), & \text{(C.29)} \\ & W_{\kappa,\mu}(z) = z^{1/2} W_{\kappa-1/2,\mu+1/2}(z) + \Big(\frac{1}{2}-\kappa-\mu\Big) W_{\kappa-1,\mu}(z) \ . & \text{(C.30)} \end{aligned}$$

Applying the formula (C.27) and its derivative to R_1 and R_3, one can prove the relation

$$R_3 = \frac{\varepsilon-4}{2} R_1 - \frac{1}{2} R_2 + R_4 \ , \tag{C.31}$$

where

$$R_4 = \int_0^1 du f_i(u)\Big[u(1-u)\Big]^{\frac{\varepsilon-2}{2}} \int_0^\infty t^{3\varepsilon/2} e^{-t+tu(1-u)/2} \partial_\xi W_{\frac{\varepsilon-4}{2},\frac{\varepsilon-3}{2}}(\xi)\, dt \ . \tag{C.32}$$

We here remove the derivative $\partial_\xi W$ from the r.h.s. of Eq.(C.32), making use of the relation (C.29):

$$R_4 = \frac{1}{2} R_2 - \frac{\varepsilon-4}{2} R_1 - - \int_0^1 du f_i(u)\Big[u(1-u)\Big]^{\frac{\varepsilon-4}{2}} \int_0^\infty t^{\frac{3}{2}\varepsilon-1} e^{-t+tu(1-u)/2} W_{\frac{\varepsilon-2}{2},\frac{\varepsilon-3}{2}}(\xi)\, dt \ . \tag{C.33}$$

Adjusting the indices on W on the r.h.s. of (C.33) in terms of the recursion relation (C.30) (with $\kappa = (\varepsilon-2)/2, \mu = (\varepsilon-3)/2$), we then apply the formula

$$\begin{aligned}\int_0^\infty t^\nu W_{\kappa,\mu}(at)e^{-pt}\,dt &= \frac{\Gamma(\nu+\mu+\frac{3}{2})\Gamma(\nu-\mu+\frac{3}{2})a^{\mu+1/2}}{\Gamma(\nu-\kappa+2)(p+a/2)^{\nu+\mu+3/2}}\\ &\times F\Big(\nu+\mu+\frac{3}{2},\mu-\kappa+\frac{1}{2};\nu-\kappa+2;\frac{2p-a}{2p+a}\Big),\\ &(\mathrm{Re}\,\nu+\frac{3}{2}-|\mathrm{Re}\,\mu|>0)\end{aligned} \tag{C.34}$$

and the r.h.s. of (C.33) becomes

$$\frac{1}{2}R_2-\frac{\varepsilon-4}{2}R_1-(3-\varepsilon)R_1-\frac{\Gamma(2\varepsilon)\Gamma(\varepsilon+2)}{\Gamma(\varepsilon+3)}\int_0^1 f_i(u)\Big[u(1-u)\Big]^{\varepsilon-2}F(2\varepsilon,1;\varepsilon+3;1-u(1-u))\ . \tag{C.35}$$

Substituting this expression for R_4 in Eq.(C.31), which should then be inserted into Eq.(C.22), and thereby eliminating R_1, we calculate C_i as follows:

$$\begin{aligned}C_i &= (m^2)^{-2\varepsilon}\int_0^1 du\, f_i(u)\Big[u(1-u)\Big]^{\varepsilon-2}\Gamma(2\varepsilon)\Big[\frac{-1}{3+\varepsilon}F(1,2\varepsilon;\varepsilon+4;1-u(1-u))\\ &+ \frac{1}{2+\varepsilon}F(1,2\varepsilon;\varepsilon+3;1-u(1-u))\Big]\\ &= \frac{(m^2)^{-2\varepsilon}\Gamma(2\varepsilon)}{4(2+\varepsilon)(3+\varepsilon)}\int_0^1 g_i(y)(1-y)^{-\frac{1}{2}}(\frac{y}{4})^{\varepsilon-2}\ F(2,2\varepsilon;\varepsilon+4;1-\frac{y}{4})\,dy\ ,\end{aligned} \tag{C.36}$$

where we have used the transformations (C.11) and (C.16) at the second equality. We now transform the argument of the hypergeometric function from $1-y/4$ to $y/4$ through the formula (C.8), and thus giving rise to $F(4-\varepsilon,2+\varepsilon;3-\varepsilon;y/4)$ and $F(2,2\varepsilon;\varepsilon-1;y/4)$. (If we apply (C.15) at this stage, we obtain the generalized hypergeometric ${}_3F_2$ expressions (5.8) and (5.9).)

For the purpose to reduce ${}_3F_2$ to ${}_2F_1$, we apply (C.16) twice to the latter F (two of them explained right above), and apply the following formula to the former F (setting $\alpha=\gamma=3-\varepsilon, \beta = 2+\varepsilon$):

$$F(\alpha+1,\beta;\gamma;z) = \frac{\beta}{\alpha}F(\alpha,\beta+1;\gamma;z)+\frac{\alpha-\beta}{\alpha}F(\alpha,\beta;\gamma;z)\ . \tag{C.37}$$

After that, we are able to integrate them by using (C.15) (with $\gamma=\nu$), and the results are therefore

$$\begin{aligned}C_3 &= (m^2)^{-2\varepsilon}\frac{1}{16}\frac{\Gamma(4+\varepsilon)\Gamma(\varepsilon-2)}{(\varepsilon+2)(\varepsilon+3)(3-\varepsilon)}\mathrm{B}(2,\frac{1}{2})\Big[\,(2+\varepsilon)F(2,3+\varepsilon;\frac{5}{2};\frac{1}{4})\\ &\quad+(1-2\varepsilon)F(2,2+\varepsilon;\frac{5}{2};\frac{1}{4})\Big]\\ &+ (m^2)^{-2\varepsilon}\Gamma(2\varepsilon)\frac{\Gamma(2-\varepsilon)}{\Gamma(4-\varepsilon)}\frac{\mathrm{B}(\frac{1}{2},\varepsilon)}{(\varepsilon-1)4^\varepsilon}\Big[\,(\varepsilon-3)F(2,2\varepsilon;\varepsilon+\frac{1}{2};\frac{1}{4})\\ &\quad+2F(3,2\varepsilon;\varepsilon+\frac{1}{2};\frac{1}{4})\Big]-C_4\ ,\end{aligned} \tag{C.38}$$

$$
\begin{aligned}
C_4 &= (m^2)^{-2\varepsilon}\frac{1}{32}\frac{\Gamma(4+\varepsilon)\Gamma(\varepsilon-2)}{(\varepsilon+2)(\varepsilon+3)(3-\varepsilon)}\mathrm{B}(3,\frac{1}{2})\Big[\,(2+\varepsilon)F(3,3+\varepsilon;\frac{7}{2};\frac{1}{4})\\
&\quad +(1-2\varepsilon)F(3,2+\varepsilon;\frac{7}{2};\frac{1}{4})\Big]\\
&+ (m^2)^{-2\varepsilon}\Gamma(2\varepsilon)\frac{\Gamma(2-\varepsilon)}{\Gamma(4-\varepsilon)}\frac{\mathrm{B}(\frac{1}{2},\varepsilon+1)}{2\varepsilon(\varepsilon-1)4^{\varepsilon}}\Big[\,(\varepsilon-3)(\varepsilon-2)F(2,2\varepsilon;\varepsilon+\frac{3}{2};\frac{1}{4})\\
&\quad +4(\varepsilon-3)F(3,2\varepsilon;\varepsilon+\frac{3}{2};\frac{1}{4})+6F(4,2\varepsilon;\varepsilon+\frac{3}{2};\frac{1}{4})\Big]\,. \qquad \text{(C.39)}
\end{aligned}
$$

D The proof of Eq.(5.14)

We show the outline of how to prove the formula (5.14). Basically we follow the same technique as we used in Appendix C, with using (C.15) and (C.16) for the aim of rearranging ${}_3F_2$ into ${}_2F_1$. We evaluate the l.h.s. of Eq. (C.15) in two ways: one is the direct result (the r.h.s. of the formula), and the other is a combination with (C.16).

On the one hand, setting $\gamma=\nu-n$ in (C.15), we have

$$
\begin{aligned}
&\int_0^1 (1-t)^{\mu-1}t^{\nu-1}F(\alpha,\beta;\nu-n;zt)\,dt = \mathrm{B}(\mu,\nu)\,{}_3F_2(\alpha,\beta,\nu;\nu-n,\mu+\nu;z),\\
&(\mathrm{Re}\,\mu,\mathrm{Re}\,\nu>0,|z|<1)\ . \qquad \text{(D.1)}
\end{aligned}
$$

On the other hand, using (C.16) $n\geq 1$ times, the integrand in Eq. (D.1) can be expanded as

$$
(1-t)^{\mu-1}t^{\nu-1}\prod_{k=1}^{n}\frac{\nu-\alpha-k}{\nu-k}\sum_{r=0}^{n}\prod_{p=1}^{r}\frac{\alpha+p-1}{\nu-\alpha-p}\binom{n}{r}\ F(\alpha+r,\beta;\nu;zt)\ , \qquad \text{(D.2)}
$$

where we define $\prod_{p=1}^{0}\equiv 1$. Then applying (C.15) (with the case $\gamma=\nu$) to this expression, the l.h.s. of (D.1) becomes

$$
\mathrm{B}(\mu,\nu)\prod_{k=1}^{n}\frac{\nu-\alpha-k}{\nu-k}\sum_{r=0}^{n}\prod_{p=1}^{r}\frac{\alpha+p-1}{\nu-\alpha-p}\binom{n}{r}\ F(\alpha+r,\beta;\mu+\nu;z)\ . \qquad \text{(D.3)}
$$

We thus have the equality

$$
{}_3F_2(\alpha,\beta,\nu;\nu-n,\mu+\nu;z)=\prod_{k=1}^{n}\frac{\nu-\alpha-k}{\nu-k}\sum_{r=0}^{n}\prod_{p=1}^{r}\frac{\alpha+p-1}{\nu-\alpha-p}\binom{n}{r}\ F(\alpha+r,\beta;\mu+\nu;z)\ . \qquad \text{(D.4)}
$$

Putting $\alpha=1$, $\nu=n+2$, and using the identity

$$
\prod_{p=1}^{r}\frac{p}{n+1-p}\binom{n}{r}=1\ , \qquad \text{(D.5)}
$$

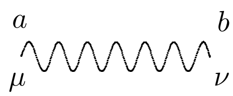
we therefore have proven the formula (5.14):

$$
{}_3F_2(1,\beta,n+2;2,\mu+\nu;z)=\frac{1}{n+1}\sum_{r=0}^{n}\ F(r+1,\beta;\mu+\nu;z)\ , \qquad \text{(D.6)}
$$

which holds for $\Re(\mu), \Re(\nu) > 0$, $|z| < 1$ and $n \geq 0$ (we have proven it for $n \geq 1$, but the $n = 0$ case is trivial).

E Feynman diagram results

In this Appendix, we present some Feynman diagram calculations in reference to the results obtained by our method. We follow the same notations as Refs. [17, 18] in the Minkowski space, however use the massive propagator in the (background) Feynman gauge:



$$= (-i)\frac{\delta^{ab} g^{\mu\nu}}{k^2 - m^2 + i\epsilon} \; .$$

We only deal with the parts which contain the "eight-figure" vacuum diagram. When the massive propagator is introduced, the tadpole contributions remain (see the diagrams (a) and (b) in Figure 2). After some calculations, the tadpole diagram (a) reads

$$\Pi_{T\ \mu\nu}^{(a)\,ab} = g^4 \frac{C_A^2 \delta^{ab}}{(4\pi)^{2-\varepsilon}} (m^2)^{1-\varepsilon} (3-2\varepsilon)\Gamma(\varepsilon-1) \Big[8(k^2 g_{\mu\nu} - k_\mu k_\nu) J_1 + D(k_\mu k_\nu J_1 + 4k_\nu J_2 + 4J_3) \Big] \; , \quad \text{(E.1)}$$

where

$$J_1 = \int \frac{d^D p}{(2\pi)^D i} \frac{1}{((p+k)^2 - m^2)(p^2 - m^2)^2}, \quad \text{(E.2)}$$

$$J_2 = \int \frac{d^D p}{(2\pi)^D i} \frac{p_\mu}{((p+k)^2 - m^2)(p^2 - m^2)^2}, \quad \text{(E.3)}$$

$$J_3 = \int \frac{d^D p}{(2\pi)^D i} \frac{p_\mu p_\nu}{((p+k)^2 - m^2)(p^2 - m^2)^2}, \quad \text{(E.4)}$$

and these are equal to

$$J_1 = -\frac{\Gamma(\varepsilon+1)}{(4\pi)^{2-\varepsilon}}\, j_1 \; , \qquad J_2 = \frac{\Gamma(\varepsilon+1)}{(4\pi)^{2-\varepsilon}} k_\mu\, j_2 \; ,$$

$$J_3 = \frac{\Gamma(\varepsilon+1)}{(4\pi)^{2-\varepsilon}} \left(\frac{1}{2\varepsilon} g_{\mu\nu}\, j_3 - \frac{1}{2} k_\mu k_\nu\, j_2\right) \; , \quad \text{(E.5)}$$

with

$$j_1 = \int_0^1 x\Big(m^2 - k^2(x - x^2)\Big)^{-1-\varepsilon} dx, \quad \text{(E.6)}$$

$$j_2 = \int_0^1 x(1-x)\Big(m^2 - k^2(x - x^2)\Big)^{-1-\varepsilon} dx, \quad \text{(E.7)}$$

$$j_3 = \int_0^1 x\Big(m^2 - k^2(x - x^2)\Big)^{-\varepsilon} = \frac{1}{2}(m^2)^{-\varepsilon} - \frac{1}{2}\varepsilon k^2 (2j_2 - j_1) \; . \quad \text{(E.8)}$$

Thus Eq.(E.1) becomes

$$\begin{aligned}\Pi^{(a)ab}_{T\ \mu\nu} &= g^4\frac{C_A^2\delta^{ab}}{(4\pi)^{4-2\varepsilon}}(m^2)^{1-\varepsilon}(3-2\varepsilon)\Gamma(\varepsilon-1)\Gamma(\varepsilon+1)\Big[\frac{D}{\varepsilon}(m^2)^{-\varepsilon} \\ &\quad +(k^2g_{\mu\nu}-k_\mu k_\nu)\Big((D-8)j_1-2Dj_2,\Big)\Big]\ ,\end{aligned} \tag{E.9}$$

and the other tadpole diagram (b) is calculated as

$$\Pi^{(b)ab}_{T\ \mu\nu} = g^4\frac{C_A^2\delta^{ab}}{(4\pi)^{4-2\varepsilon}}(m^2)^{1-2\varepsilon}g_{\mu\nu}(3-2\varepsilon)D\Gamma(\varepsilon-1)\Gamma(\varepsilon)\ . \tag{E.10}$$

As a result, the sum of these tadpole contributions takes the transversal form:

$$\begin{aligned}\Pi_{T\mu\nu}^{ab} &= \Pi^{(a)ab}_{T\ \mu\nu}+\Pi^{(b)ab}_{T\ \mu\nu} \\ &= g^4\frac{C_A^2\delta^{ab}}{(4\pi)^{4-2\varepsilon}}(m^2)^{1-\varepsilon}(3-2\varepsilon)\Gamma(\varepsilon-1)\Gamma(\varepsilon+1) \\ &\quad \times(k^2g_{\mu\nu}-k_\mu k_\nu)\Big[(D-8)j_1-2Dj_2\Big]\ .\end{aligned} \tag{E.11}$$

When $k^2\to 0$ ($|k^2| << m^2$), the quantities j_i behave as

$$j_1=\frac{1}{2}(m^2)^{-1-\varepsilon}\ ,\qquad j_2=\frac{1}{6}(m^2)^{-1-\varepsilon}\ ,\qquad j_3=\frac{1}{2}(m^2)^{-\varepsilon}\ , \tag{E.12}$$

and when $m^2\to 0$, they reduce to

$$\begin{aligned}&j_1=\frac{1}{2}(-k^2)^{-1-\varepsilon}\mathrm{B}(-\varepsilon,-\varepsilon)\ ,\qquad j_2=(-k^2)^{-1-\varepsilon}\mathrm{B}(1-\varepsilon,1-\varepsilon)\ , \\ &j_3=\frac{1}{2}(-k^2)^{-\varepsilon}\mathrm{B}(1-\varepsilon,1-\varepsilon)\ .\end{aligned} \tag{E.13}$$

Therefore in these limits Eq.(E.11) behaves

$$\Pi_{T\mu\nu}^{ab}=g_0^4\frac{C_A^2\delta^{ab}}{(4\pi)^4}(k^2g_{\mu\nu}-k_\mu k_\nu)\begin{cases}\dfrac{10}{\varepsilon} & (k^2\to 0)\\ 0 & (m=0)\end{cases}\quad +\ \mathcal{O}(1)\ . \tag{E.14}$$

Next, the diagram (c) amounts to

$$\Pi^{(e)ab}_{\ \ \mu\nu}=-6g_0^4\frac{C_A^2\delta^{ab}}{(4\pi)^4}(k^2g_{\mu\nu}-k_\mu k_\nu)\exp[2\varepsilon\ln(4\pi\mu^2)]\Gamma^2(\varepsilon)(2j_3)^2\ , \tag{E.15}$$

where j_3 is given by (E.8). Substituting Eqs. (E.12) and (E.13) for this j_3 in each limit, we derive the following limits of (E.15):

$$\Pi^{(e)ab}_{\ \ \mu\nu}=g_0^4\frac{C_A^2\delta^{ab}}{(4\pi)^4}(k^2g_{\mu\nu}-k_\mu k_\nu)\begin{cases}-\dfrac{6}{\varepsilon^2}+\dfrac{12}{\varepsilon}\rho_m & (k^2\to 0)\\ -\dfrac{6}{\varepsilon^2}-\dfrac{24}{\varepsilon}+\dfrac{12}{\varepsilon}\rho & (m=0)\end{cases}\quad +\ \mathcal{O}(1)\ , \tag{E.16}$$

where ρ_m is given by (5.19), and

$$\rho = \gamma_E + \ln \frac{-k^2}{4\pi\mu^2} \ . \tag{E.17}$$

We therefore conclude that our results coincide with the Feynman diagram calculations in the situation $k^2 \to 0$ ($m \neq 0$) (q.v. (5.23) and (5.24)):

$$\Pi_T{}^{ab}_{\mu\nu} = g_0^4 \frac{C_A^2 \delta^{ab}}{(4\pi)^4} (k^2 g_{\mu\nu} - k_\mu k_\nu)(-10C'_6) = \Pi_6{}^{ab}_{\mu\nu} \ , \tag{E.18}$$

$$\Pi^{(e)}{}^{ab}_{\mu\nu} = g_0^4 \frac{C_A^2 \delta^{ab}}{(4\pi)^4} (k^2 g_{\mu\nu} - k_\mu k_\nu)(-6C'_5) = \Pi_5{}^{ab}_{\mu\nu} \ . \tag{E.19}$$

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